Problem Set — Week 1, Session 2 · Solutions

Mathematical Statistics I · Wackerly §§1.3–1.6 · ADA University

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260912

A junior analyst records the monthly excess returns (%) of a small-cap stock over 6 months:

10, 2, 15, 3, 7, 1

Using the computational formula $s^2 = \frac{1}{n-1}\left[\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right]$, find:

(a) the sample mean ________________

(b) the sample variance ________________

(c) the sample standard deviation ________________

Solution

$\sum y_i = 38$, so $\bar{y} = 38/6 = 6.33333333333333$.

$\sum y_i^2 = 388$, so $s^2 = \dfrac{388 - 38^2/6}{6-1} = 29.4666666666667$, and
$s = \sqrt{29.4666666666667} = 5.42832079621928$.

Problem 2

seed 20260912

A quant records the absolute day-over-day price change (in cents) of a small-cap stock over 10 trading days:

51, 38, 59, 39, 46, 36, 46, 41, 24, 28

(a) Use the range (range ÷ 4) to approximate the standard deviation ________________

(b) Calculate the actual sample standard deviation ________________

Solution

Range $= 59 - 24 = 35$, so the approximation is $35/4 = 8.75$.

The actual mean is $\bar{y} = 40.8$ and $s = 10.3794669098819$. The range/4 rule is only a rough check; it tends to work best for roughly mound-shaped data and moderate sample sizes.

Problem 3

seed 20260912

An insurer records seven claims on a small book, in thousands of manat:

32, 20, 39, 21, 27, 19, 300

Give each answer to at least four significant figures.

(a) The sample mean is ______________ thousand.

(b) The sample median is ______________ thousand.

(c) The mean minus the median is ______________ thousand.

Solution

(a) $\bar{y} = \frac{1}{7} \sum y_i = \frac{458}{7} = 65.4285714285714$.

(b) Sorted, the seven claims are 19, 20, 21, 27, 32, 39, 300. With $n = 7$ the median is the fourth value, 27.

(c) $65.4285714285714 - 27 = 38.4285714285714$.

The gap in (c) is the whole lesson. One large claim drags the **mean** a long way up while leaving the **median** where it was, because the median cares only about the middle position and not about how extreme the extremes are. A mean well above the median is the signature of a right-skewed set of measurements, and claim sizes, loan losses and trading revenues are all like this.

Which is the honest summary depends on the question. For "what will the total cost of these claims be", the mean is the right number, because the total is $n\bar{y}$. For "what does a typical claim look like", the median is, because no single claim in this book resembles 65.4285714285714.

Problem 4

seed 20260912

A fund's 270 daily percentage returns have sample mean $\bar{y} = 0.16$% and sample standard deviation $s = 1.9$%. Nothing is known about the shape of the distribution, so the empirical rule is not available.

Apply Tchebysheff's theorem with $k = 4$. Give each answer to at least four significant figures; the count in part (d) need not be a whole number.

(a) At least what **proportion** of the days must lie within k standard deviations of the mean? ______________

(b) The lower endpoint of that interval is ______________ %.

(c) The upper endpoint is ______________ %.

(d) At least how many of the 270 days must lie inside the interval? ______________

Solution

(a) Tchebysheff's theorem holds for **any** data set and any $k > 1$: at least $1 - 1/k^2$ of the measurements lie within k standard deviations of the mean.

$$1 - \frac{1}{k^2} = 1 - \frac{1}{4^2} = 0.9375$$

(b), (c) The interval is $\bar{y} \pm ks$:

$$0.16 \pm 4(1.9) = (-7.44,\ 7.76)$$

(d) $0.9375 \times 270 = 253.125$ days.

Two things the theorem does **not** say, and both matter in a risk report. It gives a **lower bound**, so for mound-shaped data the actual proportion inside is usually far higher — at $k = 2$ the guarantee is 75% where the empirical rule would suggest about 95%. And it says nothing about how the mass outside the interval splits between the two tails: in principle all of it could sit in the left one, which for a fund is precisely the tail that matters.

Problem 5

seed 20260912

A bank times 880 loan applications through its processing system. The times are mound-shaped and roughly symmetric, with mean 424 seconds and standard deviation 33 seconds.

Give each answer to at least four significant figures; the count need not be a whole number.

(a) About 68% of the times lie between ______________ and ______________ seconds.

(b) The lower endpoint of the interval holding about 95% is ______________ seconds.

(c) About how many of the 880 applications fall within two standard deviations of the mean? ______________

Solution

(a) $\bar{y} \pm s = 424 \pm 33$, giving $(391,\ 457)$ seconds.

(b) $\bar{y} - 2s = 424 - 2(33) = 358$ seconds.

(c) $0.95 \times 880 = 836$ applications.

The empirical rule is the statement that for mound-shaped, roughly symmetric data, about 68%, 95% and 99.7% of the measurements lie within one, two and three standard deviations of the mean. It is an **approximation**, not a theorem, and it is earned entirely by the shape assumption stated in the problem.

Take that assumption away and the rule has no licence. Processing times are often right-skewed — a long tail of applications that hit a manual review — and in that case the 95% figure will overstate how much of the data really sits in $\bar{y} \pm 2s$. When the shape is unknown or ugly, Tchebysheff's theorem is the tool that still applies, at the price of a much weaker bound.

Problem 6

seed 20260912

Daily settlement values at a custodian are mound-shaped and roughly symmetric, with mean 122 million manat and standard deviation 14 million. The record covers 590 days.

Give each answer to at least four significant figures; the count need not be a whole number.

(a) Two standard deviations above the mean is ______________ million.

(b) By the empirical rule, what proportion of days lie **outside** $\bar{y} \pm 2s$ altogether? ______________

(c) What proportion lie above $\bar{y} + 2s$? ______________

(d) On about how many of the 590 days did settlement exceed 150 million? ______________

Solution

(a) $122 + 2(14) = 150$ million.

(b) The empirical rule puts about 95% inside, so about $1 - 0.95 = 0.05$ lies outside.

(c) Here the **symmetry** assumption does the work: the 5% outside splits evenly between the two tails, so about $0.05/2 = 0.025$ lies above.

(d) $0.025 \times 590 = 14.75$ days.

Part (c) is the step to be careful with, and it is the one Tchebysheff's theorem cannot imitate. Tchebysheff at $k = 2$ guarantees at least 75% inside, so at most 25% outside — but it says nothing whatsoever about how that 25% divides between the tails. In principle all of it could sit in one. Splitting a tail probability in half requires symmetry, and symmetry is an assumption about the data, not a fact about arithmetic.

Problem 7

seed 20260912

An analyst holds 400 daily percentage returns with mean $\bar{y} = 0.16\%$ and standard deviation $s = 1.9\%$. She is interested in the interval $\bar{y} \pm 2s$.

Give each answer to at least four significant figures; the counts need not be whole numbers.

(a) How many of the 400 days does **Tchebysheff's theorem** guarantee lie inside that interval? ______________

(b) If the returns were mound-shaped and roughly symmetric, how many would the **empirical rule** put inside it?

____

(c) By how many days do the two figures differ? ______________

Solution

(a) With $k = 2$, Tchebysheff guarantees at least $1 - 1/2^2 = 0.75$, so at least $0.75 \times 400 = 300$ days.

(b) The empirical rule puts about 95% inside, so about $0.95 \times 400 = 380$ days.

(c) $380 - 300 = 80$ days.

The gap is the price of the assumption, and it is worth being clear about which direction each statement can fail in. Tchebysheff's 75% is a **lower bound** that holds for any distribution whatsoever — it cannot be wrong, only loose. The empirical rule's 95% is an **approximation** that can be wrong in either direction, and is wrong specifically when the data are not mound-shaped.

Daily equity returns are famously heavy-tailed: too much mass in the middle and too much far out in the tails, both at once. For such data the true coverage of $\bar{y} \pm 2s$ typically falls short of 95%, so quoting the empirical rule understates how often large losses occur — and the direction of that error is against you.

Problem 8

seed 20260912

A colleague has already summarised a sample of $n = 10$ monthly fee revenues (in thousands of manat) and gives you only the two sums:

$$\sum_{i=1}^{10} y_i = 563, \qquad \sum_{i=1}^{10} y_i^2 = 34599$$

The individual figures have been lost. Give each answer to at least four significant figures.

(a) The sample mean is ________________

(b) The sample variance is ________________

(c) The sample standard deviation is ________________

Solution

(a) $\bar{y} = 563/10 = 56.3$.

(b) The defining formula needs every y_i, which you do not have. The **computing formula** needs only the two sums:

$$s^2 = \frac{1}{n-1}\left[\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right] = \frac{1}{10-1}\left[34599 - \frac{563^2}{10}\right] = 322.455555555555$$

(c) $s = \sqrt{322.455555555555} = 17.9570475177729$.

This is why the computing formula is worth knowing rather than merely deriving: it turns the variance into a function of two running totals, so a single pass through the data suffices and the raw values need not be kept. That is exactly how a database or a spreadsheet computes it over millions of rows.

One warning about it, though. When the y_i are large numbers of similar size — index **levels** rather than returns — the two terms inside the bracket very nearly cancel, and the subtraction loses most of the significant digits it started with. On such data the defining formula, computed in two passes, is the numerically safer route. Work with returns, not with prices.

Problem 9

seed 20260912

A risk report gives the daily profit and loss of a desk as having mean 65 thousand manat and **variance** 184 thousand manat squared.

Give each answer to at least four significant figures.

(a) The standard deviation is ______________ thousand manat.

(b) The lower endpoint of $\bar{y} - 2s$ is ______________ thousand.

(c) The upper endpoint $\bar{y} + 2s$ is ______________ thousand.

Solution

(a) $s = \sqrt{s^2} = \sqrt{184} = 13.5646599662505$ thousand manat.

(b), (c) $65 \pm 2(13.5646599662505)$, giving $(37.8706800674989,\ 92.1293199325011)$ thousand.

The reason the standard deviation exists at all, rather than the course simply using the variance, is the units. The variance came out in **thousands of manat squared** — a quantity with no interpretation anyone can hold in their head, and one that cannot be added to or subtracted from a mean measured in thousands of manat. Taking the square root returns a spread in the same units as the data, which is what makes an interval like $\bar{y} \pm 2s$ meaningful.

Watch for the trap in a report that names one and means the other. An interval built from the variance instead of the standard deviation would here run from $65 - 2(184)$ upward — wrong by a factor of s, and wrong in a direction that makes the desk look far riskier than it is.

Problem 10

seed 20260912

Six monthly returns have been converted to deviations from their own sample mean, $y_i - \bar{y}$. Five of the six are

$$-3, \quad -5, \quad 7, \quad 5, \quad 6$$

and the sixth has been lost. Give each answer to at least four significant figures.

(a) What must the sixth deviation be? ______________

(b) The sample variance s^2 is ______________

(c) The sample standard deviation is ______________

(d) What value would you have got in (b) by dividing by n instead of $n - 1$? ______________

Solution

(a) Deviations from the sample mean always sum to zero:

$$\sum_{i=1}^{n}(y_i - \bar{y}) = \sum y_i - n\bar{y} = n\bar{y} - n\bar{y} = 0$$

So the missing one is forced: $-(-3 - 5 + 7 + 5 + 6) = -10$.

(b) $s^2 = \frac{1}{n-1}\sum(y_i - \bar{y})^2 = \frac{244}{5} = 48.8$.

(c) $s = 6.98569967862919$.

(d) Dividing by $n = 6$ would give $244/6 = 40.6666666666667$, which is smaller.

Part (a) **is** the reason for the $n - 1$. Fix any five of the six deviations and the sixth is determined — there are only **five degrees of freedom** among them, so the average squared deviation is taken over five, not six. Dividing by n would systematically understate the spread, exactly as (d) shows, and the understatement is by the factor $(n-1)/n$ every time, not occasionally.

Chapter 8 turns this into the precise statement that $E(s^2) = \sigma^2$ when the divisor is $n - 1$: the estimator is unbiased. For now the counting argument is enough, and it is worth being able to reproduce in one line.

Problem 11

seed 20260912

Monthly gross fees across a bank's accounts have mean 101 manat and standard deviation 16 manat. Every account is then charged the **same** fixed maintenance fee of 25 manat, and the results are finally reported in a foreign currency at 2.7 manat per unit.

Give each answer to at least four significant figures.

(a) The mean net fee, in manat, is ________________

(b) The standard deviation of the net fee, in manat, is ________________

(c) The mean net fee, in the foreign currency, is ________________

(d) Its standard deviation, in the foreign currency, is ________________

Solution

Write the transformation as $z_i = (y_i - c)/r$ with $c = 25$ and $r = 2.7$.

(a) Subtracting the same constant from every observation shifts the mean by that constant: $101 - 25 = 76$ manat.

(b) It does **not** change the spread at all. Every deviation $y_i - \bar{y}$ is unchanged, because both the observation and the mean moved by the same amount, so $s = 16$ manat still.

(c) Dividing every observation by 2.7 divides the mean by it: $76/2.7 = 28.1481481481481$.

(d) And divides the standard deviation by it too: $16/2.7 = 5.92592592592593$.

The pattern is worth memorising because it is the whole of the reasoning behind standardisation. Under $z = (y - c)/r$: the mean transforms exactly as the data do, while the standard deviation responds only to the **scale** r and ignores the **shift** c. A measure of spread that changed when you moved the origin would be a bad measure of spread.

Choose $c = \bar{y}$ and $r = s$ and you get the standardised values, with mean 0 and standard deviation 1 — the transformation that makes Tchebysheff's theorem and the empirical rule statements about k alone, with no units left in them.

Problem 12

seed 20260912

Days-to-settle for a portfolio of trades are recorded only in grouped form:

Days to settle, grouped

Class	Midpoint	Frequency
15 up to 29	22	24
29 up to 43	36	26
43 up to 57	50	31
57 up to 71	64	13

Treat every observation in a class as sitting at that class's midpoint. Give each answer to at least four significant figures.

(a) How many trades are there? ______________

(b) The grouped-data mean is ______________ days.

(c) The grouped-data variance is ______________

(d) The grouped-data standard deviation is ______________ days.

Solution

(a) $n = 24 + 26 + 31 + 13 = 94$.

(b) Each midpoint is weighted by its frequency:

$$\bar{y} = \frac{\sum f_i m_i}{n} = \frac{3846}{94} = 40.9148936170213$$

(c) The same weighting inside the sum of squares:

$$s^2 = \frac{\sum f_i (m_i - \bar{y})^2}{n-1} = \frac{18701.3191489362}{94-1} = 201.089453214367$$

(d) $s = 14.1806012994643$ days.

Every number here is an **approximation**, and the assumption behind it is stated in the problem: the observations in a class are treated as if they all sat at the midpoint. That is exactly false — they are spread across the class — and the effect is to understate the true spread slightly, because the within-class variation has been thrown away.

The approximation is good when the classes are narrow relative to the overall spread, and poor when they are wide. It is what you are forced into whenever the raw data have been discarded and only the table survives, which in practice is most published data.

Problem 13

seed 20260912

Daily margin calls at a clearing member have mean 226 thousand manat and standard deviation 23 thousand. The distribution's shape is unknown, so only Tchebysheff's theorem is available.

The risk committee wants an interval, centred on the mean, that is guaranteed to contain at least 0.95 of the days. Give each answer to at least four significant figures.

(a) What value of k is required? ________________

(b) The lower endpoint of the interval is ________________ thousand.

(c) The upper endpoint is ________________ thousand.

(d) How wide is the interval? ________________ thousand.

Solution

(a) Tchebysheff guarantees at least $1 - 1/k^2$ inside $\bar{y} \pm ks$. Set that equal to the required coverage and solve:

$$1 - \frac{1}{k^2} = 0.95 \quad\Longrightarrow\quad k^2 = \frac{1}{1 - 0.95} \quad\Longrightarrow\quad k = 4.47213595499958$$

(b), (c) $226 \pm 4.47213595499958(23)$, giving $(123.14087303501,\ 328.85912696499)$ thousand.

(d) $2ks = 205.718253929981$ thousand.

Notice how expensive certainty is here. Going from a 75% guarantee ($k = 2$) to a 96% guarantee ($k = 5$) does not cost 28% more width — it costs **two and a half times** the width, because k grows like $1/\sqrt{1-p}$. Demanding a guarantee close to 1 pushes the interval out towards uselessness.

That is the standing trade-off of a distribution-free bound. If the committee were willing to assume a mound shape, the empirical rule would deliver about 95% at $k = 2$ — the same coverage for a fraction of the width. The extra width is precisely what is being paid for the freedom to say nothing about the shape.

Problem 14

seed 20260912

A fund has 344 daily returns with mean 56 basis points and standard deviation 18 basis points. Counting directly, 299 of the days fall inside $\bar{y} \pm 2s$.

Give each answer to at least four significant figures.

(a) The lower endpoint of $\bar{y} \pm 2s$ is ______________ basis points.

(b) How many days fall **outside** the interval? ______________

(c) What proportion of the days actually fall inside? ______________

(d) By how much does the actual proportion exceed Tchebysheff's guarantee at $k = 2$? ______________

Solution

(a) $56 - 2(18) = 20$ basis points.

(b) $344 - 299 = 45$ days.

(c) $299/344 = 0.869186046511628$.

(d) Tchebysheff at $k = 2$ guarantees at least $1 - 1/4 = 0.75$, so the excess is $0.869186046511628 - 0.75 = 0.119186046511628$.

The answer to (d) is positive, and it had to be. Tchebysheff's theorem is a **lower bound** that holds for every data set without exception, so an actual proportion below 0.75 would not be evidence against the theorem — it would be evidence of an arithmetic error in $\bar{y}$, s, or the count.

That is how to use the theorem in practice: as a check on your own work first, and only then as a statement about the data. And the size of the excess is informative in its own right. An actual coverage near 0.95 suggests the returns are roughly mound-shaped, in which case the empirical rule would have been the sharper tool; an actual coverage close to 0.75 says the distribution has fat tails and Tchebysheff is doing real work.

Problem 15

seed 20260912

Two branches report average transaction sizes. Branch A: 49 transactions averaging 89 manat. Branch B: 35 transactions averaging 64 manat.

Give each answer to at least four significant figures; (d) may be negative.

(a) What is the total value of branch A's transactions? ______________ manat.

(b) What is the average transaction size across both branches combined? ______________ manat.

(c) What would you get by simply averaging the two branch averages? ______________ manat.

(d) The correct figure minus that simple average is ______________ manat.

Solution

(a) A mean is a total divided by a count, so the total is the mean times the count: $49 \times 89 = 4361$ manat.

(b) Rebuild the combined mean from totals, not from means:

$$\bar{y} = \frac{n_1\bar{y}_1 + n_2\bar{y}_2}{n_1 + n_2} = \frac{4361 + 2240}{84} = 78.5833333333333$$

(c) $(89 + 64)/2 = 76.5$ manat.

(d) $78.5833333333333 - 76.5 = 2.08333333333333$ manat.

The combined mean is a **weighted** average, each branch weighted by its own sample size. The unweighted average in (c) gives the two branches equal say regardless of how many transactions stand behind each, so it agrees with the correct answer only when $n_1 = n_2$ and otherwise pulls towards whichever branch has fewer observations.

This is not a small technicality. It is the mechanism behind Simpson's paradox, and it is the single most common way a report that aggregates desks, regions or periods of unequal size arrives at a number nobody can reproduce.

Problem 16

`seed 20260912`

Weekly operating costs for a proprietary trading desk (in $ thousands), adjusted for inflation, are approximately normally distributed with mean $\mu = 530$ and standard deviation $\sigma = 35$. Next week's budget is set exactly 2 standard deviation(s) above the mean.

(a) The budgeted figure is ________________ ($ thousands).

(b) Using the empirical rule, what is the approximate probability that actual cost exceeds this budgeted figure? ________________ %

Solution

Budget $= \mu + 2\sigma = 530 + 2(35) = 600$.

By the empirical rule, $\mu \pm \sigma$ captures 68% (leaving 16% in each tail) and $\mu \pm 2\sigma$ captures 95% (leaving 2.5% in each tail), so the probability of exceeding the budget is 2.5%.

Problem 17

`seed 20260912`

Credit default swap (CDS) spreads (in basis points) for 8 corporate bonds are:

220, 160, 255, 165, 195, 155, 200, 175

(a) Sample mean ________________

(b) Sample standard deviation ________________

(c) How many of the 8 spreads fall within 1 standard deviation of the mean? ________________

(Compare this to the empirical rule's prediction of about 68% of 8, roughly 0.68×8.)

Solution

$\bar{y} = 190.625$, $s = 34.2717605533852$. The interval $\bar{y} \pm s = (156.353239446615, 224.896760553385)$ contains 6 of the 8 spreads. With only 8 observations, some deviation from the empirical rule's 68% guideline is expected.

Problem 18

seed 20260912

Risk scores for 500 advisory clients exhibit a bell-shaped (approximately normal) distribution with mean 59 and standard deviation 10.

(a) Approximately how many clients have a risk score within 1 standard deviation of the mean? ______________

(b) Approximately how many clients have a risk score within 2 standard deviations of the mean?

____

Solution

By the empirical rule, about 68% of 500 clients, or $0.68 \times 500 = 340$, fall within $\mu \pm \sigma$, and about 95%, or $0.95 \times 500 = 475$, fall within $\mu \pm 2\sigma$.

Problem 19

seed 20260912

A volatile small-cap stock had the following daily returns (in basis points) over 15 trading days:

61, 23, 87, 27, 48, 19, 48, 33, -17, -6, 50, -25, 17, 16, -18

(a) Approximate the standard deviation using the range/4 rule. ______________

(b) Calculate the actual sample mean ______________ and standard deviation ______________.

(c) Use Tchebysheff's theorem with $k = 2$ to find values a and b such that at least 75% of the returns lie between them: $a =$ ______________, $b =$ ______________

(d) What percentage of the 15 actual returns fall between a and b? ______________ % (Confirm that this actual percentage is at least 75%, as Tchebysheff's theorem guarantees.)

Solution

Range $= 87 + 25 = 112$, so the rough approximation is $112/4 = 28$.

Actual values: $\bar{y} = 24.2$, $s = 31.780946672945$.

Tchebysheff bounds ($k = 2$): $a = \bar{y} - 2s = -39.3618933458899$, $b = \bar{y} + 2s = 87.7618933458899$.

Counting the data in $[a, b]$ gives 15 of 15, or 100%, which meets or exceeds the theorem's guaranteed minimum of 75%.

Problem 20

seed 20260912

A desk's daily returns over 12 trading days, in basis points, were

51, 19, 72, 23, 40, 16, 40, 28, -14, -5, 42, -21

Give each answer to at least four significant figures.

(a) The sample mean is ______________ basis points.

(b) The sample standard deviation is ______________ basis points.

(c) The range approximation $\text{range}/4$ gives ______________

(d) The lower endpoint of $\bar{y} \pm 2s$ is ______________

(e) What proportion of the 12 returns actually fall inside $\bar{y} \pm 2s$? ______________

Solution

(a) $\bar{y} = 291/12 = 24.25$ basis points.

(b) $s^2 = \frac{1}{n-1} \sum(y_i - \bar{y})^2 = \frac{8284.25}{12-1} = 753.113636363636$, so $s = 27.442915959563$.

(c) The range is $72 - (-21) = 93$, so $\text{range}/4 = 23.25$.

(d) $24.25 - 2(27.442915959563) = -30.635831919126$.

(e) Counting, 12 of the 12 returns lie inside, so the actual proportion is 1.

Three checks to run on your own work, in this order. First, compare (c) with (b): the range approximation should land in the same neighbourhood as s, and if it is off by a factor of ten you have made an arithmetic slip rather than discovered something about the data. Second, confirm that (e) is at least 0.75 — Tchebysheff's theorem guarantees it for **any** data set, so a smaller value means a mistake in $\bar{y}$, s or the count, not a counter-example.

Third, and more interesting: (e) is typically well above 0.75 and often near 1 for a sample this small, which shows how loose the distribution-free bound is in practice. Looseness is the price of a guarantee that never fails, and deciding whether to pay it is the judgement this session is really about.

Answer key

Problem	Answers
1	(a) 6.33333, (b) 29.4667, (c) 5.42832
2	(a) 8.75, (b) 10.3795
3	(a) 65.4286, (b) 27, (c) 38.4286
4	(a) 0.9375, (b) -7.44, (c) 7.76, (d) 253.125
5	(a) 391, (b) 457, (c) 358, (d) 836

6	(a) 150, (b) 0.05, (c) 0.025, (d) 14.75
7	(a) 300, (b) 380, (c) 80
8	(a) 56.3, (b) 322.456, (c) 17.957
9	(a) 13.5647, (b) 37.8707, (c) 92.1293
10	(a) -10, (b) 48.8, (c) 6.9857, (d) 40.6667
11	(a) 76, (b) 16, (c) 28.1481, (d) 5.92593
12	(a) 94, (b) 40.9149, (c) 201.089, (d) 14.1806
13	(a) 4.47214, (b) 123.141, (c) 328.859, (d) 205.718
14	(a) 20, (b) 45, (c) 0.869186, (d) 0.119186
15	(a) 4361, (b) 78.5833, (c) 76.5, (d) 2.08333
16	(a) 600, (b) 2.5
17	(a) 190.625, (b) 34.2718, (c) 6
18	(a) 340, (b) 475
19	(a) 28, (b) 24.2, (c) 31.7809, (d) -39.3619, (e) 87.7619, (f) 100
20	(a) 24.25, (b) 27.4429, (c) 23.25, (d) -30.6358, (e) 1