

Problem Set — Week 2, Session 1

Mathematical Statistics I · Wackerly §§2.1–2.5 · ADA University

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260916

A fund holds 79 bonds. Let A be the event that a bond drawn from the portfolio is investment grade and B the event that it is denominated in AZN. Of the 79 bonds, 39 are investment grade, 26 are denominated in AZN, and 17 are both.

Give each answer as a **number of bonds**.

(a) $A \cup B$ contains _____ bonds.

(b) $A \cap \bar{B}$ contains _____ bonds.

(c) $\bar{A} \cap \bar{B}$ contains _____ bonds.

(d) $\bar{A}$ contains _____ bonds.

Problem 2

seed 20260916

A research desk covered 109 stocks last quarter and issued exactly one recommendation for each: 42 **buy**, 27 **hold**, and 40 **sell**. One covered stock is selected at random. Let A be the event that it was rated buy, B that it was rated hold, and C that it was rated sell.

Give probabilities to at least 3 decimal places.

(a) $P(A) =$ _____

(b) $P(A \cup B) =$ _____

(c) $P(\bar{C}) =$ _____

Problem 3

seed 20260916

An experiment consists of recording which of five trading strategies a fund follows in a given month. The sample space is $S = \{E_1, E_2, E_3, E_4, E_5\}$, and the probabilities of four of the five simple events are

$P(E_1) = 0.1$, $P(E_2) = 0.08$, $P(E_3) = 0.16$, $P(E_4) = 0.12$.

Give answers to at least 3 decimal places.

(a) $P(E_5) =$ _____

(b) Let $A = \{E_1, E_3\}$. Then $P(A) =$ _____

(c) Let $B = \{E_2, E_4, E_5\}$. Then $P(B) =$ _____

seed 20260916

Problem 4

A bank's loan book contains 438 loans: 19 are in default, 22 are on the watch list but not in default, and 397 are performing normally. One loan file is pulled at random, so that every loan is equally likely to be chosen.

Give probabilities to at least 3 decimal places.

(a) $P(\text{in default}) =$ _____

(b) $P(\text{performing normally}) =$ _____

(c) $P(\text{not performing normally}) =$ _____

seed 20260916

Problem 5

Over one year a corporate bond's rating changes in exactly one of five ways. The simple events and their probabilities are

E_1 : upgraded two notches, $P(E_1) = 0.07$

E_2 : upgraded one notch, $P(E_2) = 0.12$

E_3 : unchanged, $P(E_3) = 0.33$

E_4 : downgraded one notch, $P(E_4) = 0.14$

E_5 : downgraded two or more notches, $P(E_5) = 0.34$

Give answers to at least 3 decimal places.

(a) A : the rating improves. $P(A) =$ _____

(b) B : the rating deteriorates. $P(B) =$ _____

(c) C : the rating changes at all. $P(C) =$ _____

seed 20260916

Problem 6

On any given trading day, let A be the event that the AZN strengthens against the dollar and B the event that the Baku bourse index closes higher. Historical records give

$P(A) = 0.35$, $P(B) = 0.26$, $P(A \cap B) = 0.16$.

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ _____

(b) $P(\bar{A} \cap \bar{B}) =$ _____

Problem 7

seed 20260916

A pension fund reviews two holdings each quarter. Let A be the event that holding 1 beats its benchmark and B the event that holding 2 beats its benchmark, with

$$P(A) = 0.33, P(B) = 0.25, P(A \cap B) = 0.14.$$

Give answers to at least 3 decimal places.

(a) $P(A \cap \bar{B}) =$ _____

(b) $P(\bar{A} \cap B) =$ _____

(c) The probability that **exactly one** of the two beats its benchmark is _____

Problem 8

seed 20260916

A portfolio of 120 corporate bonds is classified by sector and by credit grade:

	Investment grade	High yield
Energy:	26	13
Other:	60	21

One bond is drawn at random. Let E be the event that it is an energy bond and G the event that it is investment grade.

(a) How many bonds are in the event E ? _____

Give the remaining answers to at least 3 decimal places.

(b) $P(E \cap G) =$ _____

(c) $P(E) =$ _____

(d) $P(E \cup G) =$ _____

Problem 9

seed 20260916

A supervisor reviewed 167 banks and flagged each for any of three concerns: A thin capital, B concentrated lending, C weak liquidity. The counts falling in each region were

only A : 28, only B : 19, only C : 37,

A and B but not C : 14, A and C but not B : 16, B and C but not A : 11,

all three: 3, and none of the three: 39.

One reviewed bank is chosen at random. Give answers to at least 3 decimal places.

(a) $P(A) =$ _____

(b) $P(A \cap B) =$ _____

(c) $P(A \cup B \cup C) =$ _____

Problem 10

seed 20260916

Over 194 trading days the Baku bourse index closed higher on 87 days, unchanged on 23 days, and lower on the remaining days.

Using the relative frequency of each outcome as an estimate of its probability, give answers to at least 3 decimal places.

(a) The estimated probability that the index closes higher is _____

(b) The estimated probability that it closes lower is _____

(c) The estimated probability that it does not close unchanged is _____

Problem 11

seed 20260916

Let A be the event that a commodity fund posts a positive month and B the event that the manat appreciates that month, with

$$P(A) = 0.4, P(B) = 0.31, P(A \cap B) = 0.21.$$

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ _____

(b) $P(\bar{A} \cup \bar{B}) =$ _____

(c) $P(\bar{A} \cap \bar{B}) =$ _____

Problem 12

seed 20260916

Let A be the event that a loan applicant is approved and B the event that an applicant is approved **at the lowest offered rate**. Every applicant approved at the lowest rate is of course approved, so $B \subset A$. Suppose

$$P(A) = 0.49 \text{ and } P(B) = 0.09.$$

Give answers to at least 3 decimal places.

(a) $P(A \cap B) =$ _____

(b) $P(A \cup B) =$ _____

(c) $P(A \cap \bar{B}) =$ _____

Problem 13

seed 20260916

A fund's quarterly outcome is recorded as exactly one of four simple events: E_1 (large loss), E_2 (small loss), E_3 (small gain), E_4 (large gain).

An analyst proposes

$$P(E_1) = 0.13, P(E_2) = 0.12, P(E_3) = 0.16, P(E_4) = 0.69.$$

(a) These four numbers sum to 1.10, so the assignment violates an axiom. What value must $P(E_4)$ take instead?

(b) With that correction, $P(E_1 \cup E_4) =$ _____

Problem 14

seed 20260916

An experiment records whether a stock closes up (U) or down (D) on each of two consecutive days. The sample space has four sample points, with probabilities

$$E_1 = (U, U): 0.21 \quad E_2 = (U, D): 0.16$$

$$E_3 = (D, U): 0.23 \quad E_4 = (D, D): 0.4$$

Give answers to at least 3 decimal places.

(a) $P(\text{exactly one up day}) =$ _____

(b) $P(\text{at least one up day}) =$ _____

(c) $P(\text{the first day is up}) =$ _____

Problem 15

seed 20260916

An insurer classifies each motor claim into exactly one severity band:

$$\text{under 1 000 AZN: } 0.33 \quad 1\,000 \text{ to } 5\,000: 0.2$$

$$5\,000 \text{ to } 20\,000: 0.17 \quad \text{over } 20\,000: 0.3$$

One claim is selected at random. Give answers to at least 3 decimal places.

(a) $P(\text{claim is under 5 000 AZN}) =$ _____

(b) $P(\text{claim is 5 000 AZN or more}) =$ _____

(c) $P(\text{claim is 1 000 AZN or more}) =$ _____

Problem 16

seed 20260916

Let A be the event that a start-up secures bank credit this year and B the event that it secures equity investment. A survey of the sector gives

$$P(A) = 0.34, P(B) = 0.29, \text{ and } P(\bar{A} \cap \bar{B}) = 0.43 \text{ — that is, } 0.43 \text{ is the probability a firm raises neither.}$$

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ _____

(b) $P(A \cap B) =$ _____

(c) $P(A \cap \bar{B}) =$ _____

Problem 17

seed 20260916

A leasing company's 171 contracts are each denominated in exactly one currency: 76 in AZN, 43 in USD and 52 in EUR. One contract is drawn at random. Let A , U and E be the events that it is denominated in AZN, USD and EUR respectively.

Give answers to at least 3 decimal places.

(a) $P(A) =$ _____

(b) $P(U \cup E) =$ _____

(c) $P(\bar{E}) =$ _____

Problem 18

seed 20260916

A regulator will award one licence, choosing at random among the bidders in an open tender. 7 bidders are domestic, 4 are from the region, and 15 are from further afield. The experiment is "observe which bidder receives the licence", and every bidder is equally likely to win.

(a) How many sample points does the sample space S contain? _____

Give the remaining answers to at least 3 decimal places.

(b) Let L be the event that a domestic bidder wins. $P(L) =$ _____

(c) $P(\bar{L}) =$ _____

(d) Let R be the event that a regional bidder wins. $P(L \cup R) =$ _____

Problem 19

seed 20260916

A development bank processed a batch of credit applications, classified by borrower type and outcome:

	Approved	Declined
SME: 29	15	
Corporate: 40		16

One application is drawn at random. Let M be the event that the borrower is an SME and A the event that the application was approved.

(a) How many sample points are in S ? _____

Give the remaining answers to at least 3 decimal places.

(b) $P(M \cap A) =$ _____

