Problem Set — Week 2, Session 1 · Solutions

Mathematical Statistics I · Wackerly §§2.1–2.5 · ADA University

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260916

A fund holds 79 bonds. Let A be the event that a bond drawn from the portfolio is investment grade and B the event that it is denominated in AZN. Of the 79 bonds, 39 are investment grade, 26 are denominated in AZN, and 17 are both.

Give each answer as a **number of bonds**.

(a) $A \cup B$ contains ______________ bonds.

(b) $A \cap \bar{B}$ contains ______________ bonds.

(c) $\bar{A} \cap \bar{B}$ contains ______________ bonds.

(d) $\bar{A}$ contains ______________ bonds.

Solution

(a) Counting the points of A and of B counts the 17 points of $A \cap B$ twice, so $|A \cup B| = 39 + 26 - 17 = 48$.

(b) $A \cap \bar{B}$ is A with its overlap removed: $39 - 17 = 22$.

(c) By De Morgan's law $\bar{A} \cap \bar{B} = \overline{A \cup B}$, so $79 - 48 = 31$.

(d) $79 - 39 = 40$.

Problem 2

seed 20260916

A research desk covered 109 stocks last quarter and issued exactly one recommendation for each: 42 **buy**, 27 **hold**, and 40 **sell**. One covered stock is selected at random. Let A be the event that it was rated buy, B that it was rated hold, and C that it was rated sell.

Give probabilities to at least 3 decimal places.

(a) $P(A) =$ ______________

(b) $P(A \cup B) =$ ______________

(c) $P(\bar{C}) =$ ______________

Solution

Because every stock receives exactly one recommendation, A, B and C are pairwise mutually exclusive and together make up S.

(a) $P(A) = 42/109 = 0.3853$.

(b) A and B are mutually exclusive, so $A \cap B = \emptyset$ and $P(A \cup B) = P(A) + P(B) = (42 + 27)/109 = 0.6330$.

(c) $P(\bar{C}) = 1 - P(C) = 1 - 40/109 = 0.6330$, which is the same as $P(A \cup B)$ here because the three events exhaust S.

Problem 3

seed 20260916

An experiment consists of recording which of five trading strategies a fund follows in a given month. The sample space is $S = \{E_1, E_2, E_3, E_4, E_5\}$, and the probabilities of four of the five simple events are

$P(E_1) = 0.1$, $P(E_2) = 0.08$, $P(E_3) = 0.16$, $P(E_4) = 0.12$.

Give answers to at least 3 decimal places.

(a) $P(E_5) =$ ____________

(b) Let $A = \{E_1, E_3\}$. Then $P(A) =$ ____________

(c) Let $B = \{E_2, E_4, E_5\}$. Then $P(B) =$ ____________

Solution

(a) The simple events are mutually exclusive and their union is S, so by Axioms 2 and 3 their probabilities sum to 1. Hence $P(E_5) = 1 - (0.1 + 0.08 + 0.16 + 0.12) = 0.54$.

(b) The probability of a compound event is the sum of the probabilities of the sample points in it: $P(A) = P(E_1) + P(E_3) = 0.26$.

(c) $P(B) = P(E_2) + P(E_4) + P(E_5) = 0.74$. Note $B = \bar{A}$, so $P(A) + P(B) = 1$.

Problem 4

seed 20260916

A bank's loan book contains 438 loans: 19 are in default, 22 are on the watch list but not in default, and 397 are performing normally. One loan file is pulled at random, so that every loan is equally likely to be chosen.

Give probabilities to at least 3 decimal places.

(a) $P(\text{in default}) =$ ____________

(b) $P(\text{performing normally}) =$ ____________

(c) $P(\text{not performing normally}) =$ ______________

Solution

Each of the 438 loans is a sample point, and all are equally likely, so the probability of an event is the number of sample points in it divided by 438.

(a) $19/438 = 0.0434$.

(b) $397/438 = 0.9064$.

(c) This is the complement of the event in **(b)**, so $1 - 0.9064 = 0.0936$, which is also $(19 + 22)/438$.

Problem 5

seed 20260916

Over one year a corporate bond's rating changes in exactly one of five ways. The simple events and their probabilities are

E_1: upgraded two notches, $P(E_1) = 0.07$

E_2: upgraded one notch, $P(E_2) = 0.12$

E_3: unchanged, $P(E_3) = 0.33$

E_4: downgraded one notch, $P(E_4) = 0.14$

E_5: downgraded two or more notches, $P(E_5) = 0.34$

Give answers to at least 3 decimal places.

(a) A: the rating improves. $P(A) =$ ______________

(b) B: the rating deteriorates. $P(B) =$ ______________

(c) C: the rating changes at all. $P(C) =$ ______________

Solution

By the sample-point method, the probability of a compound event is the sum of the probabilities of the sample points it contains.

(a) $P(A) = P(E_1) + P(E_2) = 0.07 + 0.12 = 0.19$.

(b) $P(B) = P(E_4) + P(E_5) = 0.14 + 0.34 = 0.48$.

(c) $C = \bar{E}_3$, so $P(C) = 1 - 0.33 = 0.67$, which agrees with $P(A) + P(B)$ because A and B are mutually exclusive.

Problem 6

seed 20260916

On any given trading day, let A be the event that the AZN strengthens against the dollar and B the event that the Baku bourse index closes higher. Historical records give

$P(A) = 0.35$, $P(B) = 0.26$, $P(A \cap B) = 0.16$.

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ ______________

(b) $P(\bar{A} \cap \bar{B}) =$ ______________

Solution

(a) Adding $P(A)$ and $P(B)$ counts $A \cap B$ twice, so subtract it once: $P(A \cup B) = 0.35 + 0.26 - 0.16 = 0.45$.

(b) By De Morgan's law $\bar{A} \cap \bar{B} = \overline{A \cup B}$, so $P(\bar{A} \cap \bar{B}) = 1 - 0.45 = 0.55$.

Problem 7

seed 20260916

A pension fund reviews two holdings each quarter. Let A be the event that holding 1 beats its benchmark and B the event that holding 2 beats its benchmark, with

$P(A) = 0.33$, $P(B) = 0.25$, $P(A \cap B) = 0.14$.

Give answers to at least 3 decimal places.

(a) $P(A \cap \bar{B}) =$ ______________

(b) $P(\bar{A} \cap B) =$ ______________

(c) The probability that **exactly one** of the two beats its benchmark is ______________

Solution

The event A splits into the part inside B and the part outside it, and these two are mutually exclusive: $P(A) = P(A \cap B) + P(A \cap \bar{B})$.

(a) $P(A \cap \bar{B}) = 0.33 - 0.14 = 0.19$.

(b) $P(\bar{A} \cap B) = 0.25 - 0.14 = 0.11$.

(c) "Exactly one" is the union of those two mutually exclusive events, so $0.19 + 0.11 = 0.3$. Equivalently $P(A) + P(B) - 2P(A \cap B)$ — the intersection is removed twice, once from each.

Problem 8

seed 20260916

A portfolio of 120 corporate bonds is classified by sector and by credit grade:

	Investment grade	High yield
Energy:	26	13
Other:	60	21

One bond is drawn at random. Let E be the event that it is an energy bond and G the event that it is investment grade.

(a) How many bonds are in the event E? ______________

Give the remaining answers to at least 3 decimal places.

(b) $P(E \cap G) =$ ______________

(c) $P(E) =$ ______________

(d) $P(E \cup G) =$ ______________

Solution

Every bond is one equally likely sample point, and there are 120 of them.

(a) $|E| = 26 + 13 = 39$.

(b) $E \cap G$ is the single cell "energy and investment grade": $26/120 = 0.2167$.

(c) $P(E) = 39/120 = 0.3250$.

(d) $|G| = 26 + 60 = 86$, and the additive law gives $|E \cup G| = 39 + 86 - 26 = 99$, so $P(E \cup G) = 99/120 = 0.8250$. Equivalently, count every bond except the 21 that are neither energy nor investment grade.

Problem 9

seed 20260916

A supervisor reviewed 167 banks and flagged each for any of three concerns: A thin capital, B concentrated lending, C weak liquidity. The counts falling in each region were

only A: 28, only B: 19, only C: 37,

A and B but not C: 14, A and C but not B: 16, B and C but not A: 11,

all three: 3, and none of the three: 39.

One reviewed bank is chosen at random. Give answers to at least 3 decimal places.

(a) $P(A) =$ ______________

(b) $P(A \cap B) =$ ______________

(c) $P(A \cup B \cup C) =$ _______________

Solution

The eight regions are mutually exclusive and exhaust S, so every event is the sum of the regions inside it — no inclusion-exclusion bookkeeping is needed once the regions are given.

(a) A is made of "only A", "A and B", "A and C" and "all three": $28 + 14 + 16 + 3 = 61$, so $P(A) = 0.3653$.

(b) $A \cap B$ contains the banks flagged for both, whether or not they were also flagged for C: $14 + 3 = 17$, so $P(A \cap B) = 0.1018$.

(c) The union is everything except the 39 banks flagged for nothing: $167 - 39 = 128$, so $P(A \cup B \cup C) = 0.7665$.

Problem 10

seed 20260916

Over 194 trading days the Baku bourse index closed higher on 87 days, unchanged on 23 days, and lower on the remaining days.

Using the relative frequency of each outcome as an estimate of its probability, give answers to at least 3 decimal places.

(a) The estimated probability that the index closes higher is _______________

(b) The estimated probability that it closes lower is _______________

(c) The estimated probability that it does not close unchanged is _______________

Solution

The relative frequency of an outcome is the number of times it occurred divided by the number of trials, and it is what the probability of that outcome is estimated by when the experiment is repeated many times.

(a) $87/194 = 0.4485$.

(b) Lower on $194 - 87 - 23 = 84$ days, so 0.4330.

(c) $(87 + 84)/194 = 0.8814$, the complement of closing unchanged.

Problem 11

seed 20260916

Let A be the event that a commodity fund posts a positive month and B the event that the manat appreciates that month, with

$P(A) = 0.4$, $P(B) = 0.31$, $P(A \cap B) = 0.21$.

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ ______________

(b) $P(\bar{A} \cup \bar{B}) =$ ______________

(c) $P(\bar{A} \cap \bar{B}) =$ ______________

Solution

(a) $0.4 + 0.31 - 0.21 = 0.5$.

(b) De Morgan: $\bar{A} \cup \bar{B} = \overline{A \cap B}$. A point is outside A or outside B exactly when it fails to be in both. So $1 - P(A \cap B) = 1 - 0.21 = 0.79$.

(c) The other law: $\bar{A} \cap \bar{B} = \overline{A \cup B}$, giving $1 - 0.5 = 0.5$.

Notice the two laws are not interchangeable — swapping them here would give the wrong answer in both parts.

Problem 12

seed 20260916

Let A be the event that a loan applicant is approved and B the event that an applicant is approved **at the lowest offered rate**. Every applicant approved at the lowest rate is of course approved, so $B \subset A$. Suppose

$P(A) = 0.49$ and $P(B) = 0.09$.

Give answers to at least 3 decimal places.

(a) $P(A \cap B) =$ ______________

(b) $P(A \cup B) =$ ______________

(c) $P(A \cap \bar{B}) =$ ______________

Solution

Because every point of B is already a point of A:

(a) $A \cap B = B$, so $P(A \cap B) = 0.09$.

(b) $A \cup B = A$, so $P(A \cup B) = 0.49$. The additive law agrees: $0.49 + 0.09 - 0.09 = 0.49$.

(c) $A \cap \bar{B}$ is the part of A outside B — approved, but not at the lowest rate: $0.49 - 0.09 = 0.4$.

Problem 13

seed 20260916

A fund's quarterly outcome is recorded as exactly one of four simple events: E_1 (large loss), E_2 (small loss), E_3 (small gain), E_4 (large gain).

An analyst proposes

$P(E_1) = 0.13$, $P(E_2) = 0.12$, $P(E_3) = 0.16$, $P(E_4) = 0.69$.

(a) These four numbers sum to 1.10, so the assignment violates an axiom. What value must $P(E_4)$ take instead? ______________

(b) With that correction, $P(E_1 \cup E_4) =$ ______________

Solution

The four simple events are mutually exclusive and their union is the whole sample space. Axiom 3 then makes $P(S)$ the sum of the four probabilities, and Axiom 2 requires $P(S) = 1$.

(a) So the four must sum to exactly 1, giving $P(E_4) = 1 - (0.13 + 0.12 + 0.16) = 0.59$.

(b) E_1 and E_4 are mutually exclusive simple events, so their probabilities add: $0.13 + 0.59 = 0.72$.

The analyst's value is not merely inaccurate: no assignment summing to more than 1 can be a probability assignment at all, whatever the data suggest.

Problem 14

seed 20260916

An experiment records whether a stock closes up (U) or down (D) on each of two consecutive days. The sample space has four sample points, with probabilities

$E_1 = (U, U)$: 0.21 $E_2 = (U, D)$: 0.16

$E_3 = (D, U)$: 0.23 $E_4 = (D, D)$: 0.4

Give answers to at least 3 decimal places.

(a) $P(\text{exactly one up day}) =$ ______________

(b) $P(\text{at least one up day}) =$ ______________

(c) $P(\text{the first day is up}) =$ ______________

Solution

Each compound event is a collection of sample points, and its probability is the sum of theirs.

(a) Exactly one up day is $\{E_2, E_3\}$: $0.16 + 0.23 = 0.39$.

(b) At least one up day is $\{E_1, E_2, E_3\}$, the complement of E_4: $1 - 0.4 = 0.6$.

(c) The first day is up in $\{E_1, E_2\}$: $0.21 + 0.16 = 0.37$.

Note that (a) and (c) are different events even though each contains two sample points — which points they contain is what matters, not how many.

Problem 15

seed 20260916

An insurer classifies each motor claim into exactly one severity band:

under 1 000 AZN: 0.33   1 000 to 5 000: 0.2

5 000 to 20 000: 0.17   over 20 000: 0.3

One claim is selected at random. Give answers to at least 3 decimal places.

(a) $P(\text{claim is under 5 000 AZN}) =$ ____________

(b) $P(\text{claim is 5 000 AZN or more}) =$ ____________

(c) $P(\text{claim is 1 000 AZN or more}) =$ ____________

Solution

The four bands are mutually exclusive and exhaust the sample space, so probabilities of unions are plain sums and the four values total 1.

(a) $0.33 + 0.2 = 0.53$.

(b) $0.17 + 0.3 = 0.47$, which is also $1 - 0.53$ since (a) and (b) are complements.

(c) The complement of the lowest band: $1 - 0.33 = 0.67$.

Problem 16

seed 20260916

Let A be the event that a start-up secures bank credit this year and B the event that it secures equity investment. A survey of the sector gives

$P(A) = 0.34$, $P(B) = 0.29$, and $P(\bar{A} \cap \bar{B}) = 0.43$ — that is, 0.43 is the probability a firm raises neither.

Give answers to at least 3 decimal places.

(a) $P(A \cup B) =$ ____________

(b) $P(A \cap B) =$ ____________

(c) $P(A \cap \bar{B}) =$ ____________

Solution

(a) "Neither" is the complement of "at least one", so $P(A \cup B) = 1 - 0.43 = 0.57$.

(b) Now run the additive law backwards. From $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
$P(A \cap B) = 0.34 + 0.29 - 0.57 = 0.06$.

(c) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.34 - 0.06$.

The order matters: the intersection cannot be read off the data directly, but it follows once the union is known.

Problem 17

seed 20260916

A leasing company's 171 contracts are each denominated in exactly one currency: 76 in AZN, 43 in USD and 52 in EUR. One contract is drawn at random. Let A, U and E be the events that it is denominated in AZN, USD and EUR respectively.

Give answers to at least 3 decimal places.

(a) $P(A) =$ ______________

(b) $P(U \cup E) =$ ______________

(c) $P(\bar{E}) =$ ______________

Solution

Each contract has exactly one currency, so A, U and E are pairwise mutually exclusive and their union is S — they partition the sample space.

(a) $76/171 = 0.4444$.

(b) Mutually exclusive, so the probabilities simply add: $(43 + 52)/171 = 0.5556$. No intersection term is subtracted, because $U \cap E = \emptyset$.

(c) $1 - P(E) = 0.6959$, the same event as $A \cup U$.

Problem 18

seed 20260916

A regulator will award one licence, choosing at random among the bidders in an open tender. 7 bidders are domestic, 4 are from the region, and 15 are from further afield. The experiment is "observe which bidder receives the licence", and every bidder is equally likely to win.

(a) How many sample points does the sample space S contain? ______________

Give the remaining answers to at least 3 decimal places.

(b) Let L be the event that a domestic bidder wins. $P(L) =$ ______________

(c) $P(\bar{L}) =$ ______________

(d) Let R be the event that a regional bidder wins. $P(L \cup R) =$ ______________

Solution

(a) Each bidder winning is one simple event that cannot be decomposed, so $|S| = 7 + 4 + 15 = 26$.

(b) L is a compound event holding 7 of those points, and the points are equally likely: $7/26 = 0.2692$.

(c) $\bar{L}$ holds the other 19: 0.7308, and $P(L) + P(\bar{L}) = 1$.

(d) L and R are mutually exclusive — one bidder wins, and no bidder is both domestic and regional — so the probabilities add: $(7 + 4)/26 = 0.4231$.

Problem 19

seed 20260916

A development bank processed a batch of credit applications, classified by borrower type and outcome:

	Approved	Declined
SME:	29	15
Corporate:	40	16

One application is drawn at random. Let M be the event that the borrower is an SME and A the event that the application was approved.

(a) How many sample points are in S? ______________

Give the remaining answers to at least 3 decimal places.

(b) $P(M \cap A) =$ ______________

(c) $P(M \cup A) =$ ______________

(d) $P(\bar{M} \cap \bar{A}) =$ ______________

(e) $P(\bar{A}) =$ ______________

Solution

Every application is one equally likely sample point, so each probability is a count over 100.

(a) $29 + 15 + 40 + 16 = 100$.

(b) $M \cap A$ is the single SME-and-approved cell: $29/100 = 0.2900$.

(c) $|M| = 44$ and $|A| = 69$, and the additive law removes the double-counted cell: $44 + 69 - 29 = 84$, so 0.8400.

(d) By De Morgan this is $\overline{M \cup A}$ — the corporate-and-declined cell, 16 applications: 0.1600. It agrees with $1 - 0.8400$.

(e) $P(\bar{A}) = 1 - 69/100 = 0.3100$, the two declined cells together.

Problem 20

seed 20260916

A fund's month is recorded as exactly one of five simple events:

E_1 severe loss: 0.1 E_2 moderate loss: 0.14 E_3 flat: 0.26

E_4 moderate gain: 0.18 E_5 large gain: ?

Let $A = \{E_1, E_2\}$ (the month is a loss) and $B = \{E_1, E_2, E_4, E_5\}$ (the month is not flat).

Give answers to at least 3 decimal places.

(a) $P(E_5) =$ ______________

(b) $P(A) =$ ______________

(c) $P(B) =$ ______________

(d) $P(A \cup B) =$ ______________

(e) $P(\bar{A} \cap \bar{B}) =$ ______________

(f) $P(B \cap \bar{A}) =$ ______________

Solution

(a) The five simple events are mutually exclusive with union S, so by Axioms 2 and 3 their probabilities sum to 1: $P(E_5) = 1 - (0.1 + 0.14 + 0.26 + 0.18) = 0.32$.

(b) $P(A) = 0.1 + 0.14 = 0.24$.

(c) $P(B) = 0.1 + 0.14 + 0.18 + 0.32 = 0.74$, equivalently $1 - P(E_3)$.

(d) Every point of A lies in B, so $A \subset B$ and $A \cup B = B$: the answer is 0.74, not the sum of the two probabilities.

(e) $\overline{A \cup B} = \bar{B}$, which is the single point E_3: $1 - 0.74 = 0.26$.

(f) $B \cap \bar{A} = \{E_4, E_5\}$: $0.18 + 0.32 = 0.5$, which is $P(B) - P(A)$ precisely because $A \subset B$.

Answer key

Problem	Answers
1	(a) 48, (b) 22, (c) 31, (d) 40
2	(a) 0.385321, (b) 0.633028, (c) 0.633028
3	(a) 0.54, (b) 0.26, (c) 0.74
4	(a) 0.043379, (b) 0.906393, (c) 0.0936073

5	(a) 0.19, (b) 0.48, (c) 0.67
6	(a) 0.45, (b) 0.55
7	(a) 0.19, (b) 0.11, (c) 0.3
8	(a) 39, (b) 0.216667, (c) 0.325, (d) 0.825
9	(a) 0.365269, (b) 0.101796, (c) 0.766467
10	(a) 0.448454, (b) 0.43299, (c) 0.881443
11	(a) 0.5, (b) 0.79, (c) 0.5
12	(a) 0.09, (b) 0.49, (c) 0.4
13	(a) 0.59, (b) 0.72
14	(a) 0.39, (b) 0.6, (c) 0.37
15	(a) 0.53, (b) 0.47, (c) 0.67
16	(a) 0.57, (b) 0.06, (c) 0.28
17	(a) 0.444444, (b) 0.555556, (c) 0.695906
18	(a) 26, (b) 0.269231, (c) 0.730769, (d) 0.423077
19	(a) 100, (b) 0.29, (c) 0.84, (d) 0.16, (e) 0.31
20	(a) 0.32, (b) 0.24, (c) 0.74, (d) 0.74, (e) 0.26, (f) 0.5