

Problem Set — Week 2, Session 2

Mathematical Statistics I · Wackerly §2.6 · ADA University

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260919

An audit record is created by naming one of **13** branches and one of **4** products. A sample point is the ordered pair (branch, product).

(a) How many such pairs are there? _____

(b) The record is later extended to name a rate tier as well, chosen from **10** tiers. How many ordered triples (branch, product, tier) are there? _____

(c) One triple is selected at random, all triples equally likely. What is the probability that a particular triple is the one selected? Give at least four significant figures. _____

Problem 2

seed 20260919

A settlement system tags every instruction with a code of **3** symbols drawn from an alphabet of **19** symbols. The code is read in order, so a sample point is an ordered **3**-tuple of symbols.

(a) If a symbol may be used more than once, how many codes are there? _____

(b) If every symbol in a code must be different, how many codes are there? _____

(c) A code is generated at random from the alphabet in (a), each of those codes equally likely. What is the probability that its **3** symbols are all different? Give at least four significant figures. _____

(d) The system it replaced used codes of **3** symbols from an alphabet of **19**, repeats allowed. How many codes did the old system have? _____

Problem 3

seed 20260919

10 research presentations are to be scheduled at a conference. No two may run at the same time, and the order in which they are given matters.

(a) In how many orders can all **10** presentations be scheduled? _____

(b) Only **3** of them will be given in the morning session, in a stated order. In how many ways can the morning session be filled? _____

(c) A schedule is drawn at random from the orderings in (a), all equally likely. What is the probability that one particular presentation is given first? Give at least four significant figures. _____

(d) Separately, **3** of **10** posters are to be hung on **3** display boards read left to right, so the order matters. In how many ways can the boards be filled? _____

Problem 4

seed 20260919

A bank's product engine builds a term-deposit configuration by choosing, in sequence, one of **6** maturities, one of **2** interest-payment frequencies, one of **9** currencies and one of **6** early-withdrawal terms. A sample point is the ordered list of the four choices.

(a) How many distinct configurations can the engine generate? _____

(b) A regulator now fixes the maturity at 12 months. How many configurations remain available?

(c) If a client picks one of the configurations in part (a) completely at random, so that the sample points are equally likely, what is the probability she picks any one particular configuration? Give at least four significant figures. _____

Problem 5

seed 20260919

A fund has **27** analysts on its research desk.

(a) **3** of them are to be given **3 different** sector mandates — each mandate distinct. How many assignments are possible? _____

(b) Instead the same **3** analysts simply form a working group of equal standing. How many groups are possible?

(c) The answer to (a) divided by the answer to (b) equals _____

(d) Separately, **3** analysts must be chosen from a shortlist of **14** for an unordered committee. How many committees are possible? _____

Problem 6

seed 20260919

A risk committee of **3** members is to be formed from **20** eligible staff. All members serve on equal terms, so a committee is an unordered subset.

(a) How many committees are possible? _____

(b) How many of them include the chief risk officer, who is one of the **20**? _____

(c) How many exclude her? _____

(d) If the committee is drawn at random, all committees equally likely, what is the probability that she is on it? Give at least four significant figures. _____

(e) Once the committee is seated, **3** of its **3** members form a drafting subcommittee, again unordered. How many subcommittees can be formed from a given committee? _____

Problem 7

seed 20260919

A working party is to be drawn from **12** actuaries and **6** lawyers. Members serve on equal terms, so a working party is an unordered subset.

- (a) In how many ways can **3** of the actuaries be chosen? _____
- (b) In how many ways can a party of **3** actuaries and **3** lawyers be formed? _____
- (c) Ignoring profession, how many parties of **6** people can be formed from the **18** candidates? _____
- (d) A party of **6** is drawn at random from all those in (c), equally likely. What is the probability that it contains exactly **3** actuaries? Give at least four significant figures. _____

Problem 8

seed 20260919

A consultant's shelf holds **12** domestic funds and **5** international funds. **5** of the **17** funds are chosen at random, without replacement, every set of **5** equally likely. A sample point is the unordered set chosen.

- (a) How many sample points does $\mathcal{S}$ contain? _____

Give the remaining answers to at least four significant figures.

- (b) $P(\text{all 5 chosen funds are domestic}) =$ _____
- (c) $P(\text{all 5 come from the same one of the two groups}) =$ _____
- (d) $P(\text{the chosen funds are not all from one group}) =$ _____

Problem 9

seed 20260919

A branch drawer holds **29** loan files, of which **2** were misreported at origination. An auditor draws **4** files at random, without replacement, and a sample point is the unordered set of files drawn.

Give the probabilities to at least four significant figures.

- (a) How many sample points does $\mathcal{S}$ contain? _____
- (b) $P(\text{exactly 2 of the drawn files are misreported}) =$ _____
- (c) $P(\text{at least one drawn file is misreported}) =$ _____

Problem 10

seed 20260919

A batch of **27** invoices contains **5** that are overdue. An auditor samples **6** invoices at random without replacement, every subset of that size equally likely.

- (a) How many sample points does $\mathcal{S}$ contain? _____

Give the remaining answers to at least four significant figures.

- (b) $P(\text{none of the sampled invoices is overdue}) =$ _____

(c) $P(\text{exactly one is overdue}) =$ _____

(d) $P(\text{at least two are overdue}) =$ _____

Problem 11

seed 20260919

A desk logs one recommendation per trading day for **15** days. Over that stretch it issued **6 buy**, **2 hold** and **7 sell** recommendations. A sample point is the sequence of **15** letters read in date order, for example B, H, S, ... Reports carrying the same recommendation are indistinguishable in the sequence.

(a) How many distinct sequences are possible? _____

(b) How many of them begin with a **buy**? _____

Give the remaining answers to at least four significant figures.

(c) One of the sequences in (a) is selected at random, all equally likely. $P(\text{it begins with a buy}) =$

(d) $P(\text{a particular sequence is the one selected}) =$ _____

Problem 12

seed 20260919

24 analysts are assigned at random to several named desks. Desk 1 has **5** places and desk 2 has **8** places; the rest are spread over the remaining desks. Every partition of the **24** analysts into the desks of these stated sizes is equally likely — the number of such partitions is the multinomial coefficient of Theorem 2.3.

3 of the analysts joined the firm together and are known as the graduate team. Give each probability to at least four significant figures.

(a) $P(\text{all 3 of the graduate team are placed on desk 1}) =$ _____

(b) $P(\text{all 3 of them are placed on desk 2}) =$ _____

(c) $P(\text{none of them is placed on desk 1}) =$ _____

Problem 13

seed 20260919

15 inspectors are to be split among three named shifts: **6** on the morning shift, **3** on the afternoon shift and **6** on the night shift. The shifts are distinguishable; the inspectors within a shift are not ranked.

(a) In how many ways can the morning shift alone be filled? _____

(b) In how many ways can the full split be made? _____

(c) One split is chosen at random, all splits equally likely. What is the probability that a particular inspector is placed on the morning shift? Give at least four significant figures. _____

(d) A second plant splits its **12** inspectors into a day panel of **5** and a night panel holding the rest. In how many ways can that split be made? _____

Problem 14

seed 20260919

3 of 18 candidates are chosen at random for a review team, every team of that size equally likely. Two of the candidates, call them Aygun and Rashad, have worked together before.

(a) How many teams are possible? _____

Give the remaining answers to at least four significant figures.

(b) $P(\text{both Aygun and Rashad are on the team}) =$ _____

(c) $P(\text{neither is on the team}) =$ _____

(d) $P(\text{exactly one of the two is on the team}) =$ _____

(e) A second round draws a team of 4 from 17 candidates. How many teams are possible there? _____

Problem 15

seed 20260919

A reference number consists of one letter chosen from 16 permitted letters followed by 2 digits, each digit from 0 to 9 and repeats allowed.

(a) How many reference numbers are there? _____

(b) 4 of the letters are reserved for internal use and may not appear on a client reference. How many client references are there? _____

(c) Returning to all 16 letters: how many reference numbers have a first digit other than 0? _____

(d) A reference number is drawn at random from those in (a), all equally likely. What is the probability it is a client reference? Give at least four significant figures. _____

Problem 16

seed 20260919

A loan agreement offers 17 optional covenants. Any collection of them may be written into the contract, including all of them and none of them. A sample point is the set of covenants included.

(a) How many different contracts can be written? _____

(b) How many contain exactly 2 covenants? _____

(c) How many contain exactly 5 covenants? _____

(d) How many contain at least one covenant? _____

(e) A contract is drawn at random from those in (a), all equally likely. What is the probability that it contains exactly 2 covenants? Give at least four significant figures. _____

Problem 17

seed 20260919

9 agenda items are to be placed in order at a board meeting. 2 of them concern the same matter and the chair would like them taken one after another.

- (a) How many orders of the 9 items are there? _____
- (b) How many orders place those 2 items in consecutive positions? _____
- (c) How many orders break them up? _____
- (d) An order is drawn at random from those in (a), all equally likely. What is the probability that the 2 items are consecutive? Give at least four significant figures. _____
- (e) At the following meeting there are 12 items and no adjacency requirement. In how many orders does one particular item come first? _____

Problem 18

seed 20260919

A portfolio holds 7 A-rated issues, 6 B-rated issues and 8 C-rated issues. 6 of the 21 issues are selected at random for review, without replacement, every set of 6 equally likely.

- (a) How many sample points does $\mathcal{S}$ contain? _____

Give the remaining answers to at least four significant figures.

- (b) $P(\text{exactly 2 A-rated, 2 B-rated and 2 C-rated are selected}) =$ _____
- (c) $P(\text{no C-rated issue is selected}) =$ _____
- (d) $P(\text{at least one C-rated issue is selected}) =$ _____

Problem 19

seed 20260919

A treasury desk holds 20 bonds. Each matures on one of 250 possible settlement dates, every date equally likely and the bonds' dates assigned independently. A sample point is an ordered 20-tuple of dates, so $\mathcal{S}$ contains 250^{20} equally likely points.

Give each probability to at least four significant figures.

- (a) $P(\text{no two bonds mature on the same date}) =$ _____
- (b) $P(\text{at least two bonds mature on the same date}) =$ _____

Problem 20

seed 20260919

A procurement office receives 12 bids. 4 of them will be shortlisted, and the shortlist is unordered.

- (a) How many shortlists are possible? _____

(b) From a given shortlist, the office will award first, second and third place, all to different bidders. In how many ways can the three places be awarded? _____

(c) Separately, **8** reviewers are split into a senior panel of **5** and a junior panel of **3**. The panels are named; the reviewers within a panel are not ranked. In how many ways can the split be made?

(d) The shortlist is drawn at random from the lists in (a), all equally likely. Two of the **12** bids came from the same parent company. What is the probability that both are shortlisted? Give at least four significant figures.