Problem Set — Week 2, Session 2 · Solutions

Mathematical Statistics I · Wackerly §2.6 · ADA University

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260919

An audit record is created by naming one of 13 branches and one of 4 products. A sample point is the ordered pair (branch, product).

(a) How many such pairs are there? ________________

(b) The record is later extended to name a rate tier as well, chosen from 10 tiers. How many ordered triples (branch, product, tier) are there? ________________

(c) One triple is selected at random, all triples equally likely. What is the probability that a particular triple is the one selected? Give at least four significant figures. ________________

Solution

(a) The mn rule (Theorem 2.1): pairing each of $m = 13$ branches with each of $n = 4$ products gives $13 \times 4 = 52$ pairs.

(b) The extension of the rule to three stages multiplies once more: $52 \times 10 = 520$.

(c) Equally likely points, so each carries probability $1/N = 1/520 = 0.00192307692307692$.

The point of the rule is that the count is available without writing the list out; the sample-point method then needs nothing else.

Problem 2

seed 20260919

A settlement system tags every instruction with a code of 3 symbols drawn from an alphabet of 19 symbols. The code is read in order, so a sample point is an ordered 3-tuple of symbols.

(a) If a symbol may be used more than once, how many codes are there? ________________

(b) If every symbol in a code must be different, how many codes are there? ________________

(c) A code is generated at random from the alphabet in (a), each of those codes equally likely. What is the probability that its 3 symbols are all different? Give at least four significant figures. ________________

(d) The system it replaced used codes of 3 symbols from an alphabet of 19, repeats allowed. How many codes did the old system have? ________________

Solution

(a) Each of the 3 positions is a stage with 19 options, and the options do not change from stage to stage, so the extended mn rule gives $19^3 = 6859$.

(b) Now each stage has one fewer option than the last, and the product $19 \times (19-1) \times \cdots$ taken over 3 factors is exactly the number of permutations of 3 symbols chosen from 19:

$$P_3^{19} = \frac{19!}{(19-3)!} = 5814$$

(c) The codes in (a) are the equally likely sample points and the codes in (b) are the ones favourable to the event, so the probability is $5814/6859 = 0.847645429362881$.

(d) The same rule with different numbers: $19^3 = 6859$.

A permutation count is nothing more than the mn rule applied when each stage consumes one of the objects.

Problem 3

seed 20260919

10 research presentations are to be scheduled at a conference. No two may run at the same time, and the order in which they are given matters.

(a) In how many orders can all 10 presentations be scheduled? ________________

(b) Only 3 of them will be given in the morning session, in a stated order. In how many ways can the morning session be filled? ________________

(c) A schedule is drawn at random from the orderings in (a), all equally likely. What is the probability that one particular presentation is given first? Give at least four significant figures. ________________

(d) Separately, 3 of 10 posters are to be hung on 3 display boards read left to right, so the order matters. In how many ways can the boards be filled? ________________

Solution

(a) Ordering all n objects is the permutation count with $r = n$: $P_{10}^{10} = 10! = 3628800$.

(b) Choosing and ordering 3 of the 10:

$$P_3^{10} = \frac{10!}{(10-3)!} = 720$$

(c) Fix the chosen presentation in the first position; the other $10-1$ may be ordered freely, which is $(10-1)!$ schedules. Dividing by $10!$ leaves $1/10 = 0.1$ — as symmetry would suggest, since no presentation is favoured over any other.

(d) $P_3^{10} = 720$ — the same count as (b) with different numbers.

Problem 4

seed 20260919

A bank's product engine builds a term-deposit configuration by choosing, in sequence, one of 6 maturities, one of 2 interest-payment frequencies, one of 9 currencies and one of 6 early-withdrawal terms. A sample point is the ordered list of the four choices.

(a) How many distinct configurations can the engine generate? ______________

(b) A regulator now fixes the maturity at 12 months. How many configurations remain available?

____

(c) If a client picks one of the configurations in part (a) completely at random, so that the sample points are equally likely, what is the probability she picks any one particular configuration? Give at least four significant figures. ______________

Solution

(a) A sample point is built by making four choices in sequence, so by the extension of Theorem 2.1 (the mn rule) we multiply the number of options available at each stage:

$$N = 6 \times 2 \times 9 \times 6 = 648$$

(b) Fixing the maturity removes the first stage entirely — one option instead of 6 — leaving

$$2 \times 9 \times 6 = 108$$

(c) With $N = 648$ equally likely sample points, each has probability $1/N = 0.00154320987654321$, directly from Axiom 3 and $P(S) = 1$.

The mn rule is what makes the sample-point method usable here: listing 648 configurations to find N would be a waste of an afternoon, and counting them takes one line.

Problem 5

seed 20260919

A fund has 27 analysts on its research desk.

(a) 3 of them are to be given 3 **different** sector mandates — each mandate distinct. How many assignments are possible? ______________

(b) Instead the same 3 analysts simply form a working group of equal standing. How many groups are possible?

____

(c) The answer to (a) divided by the answer to (b) equals ______________

(d) Separately, 3 analysts must be chosen from a shortlist of 14 for an unordered committee. How many committees are possible? ______________

Solution

(a) Distinct mandates make the selection **ordered**, so this is a permutation (Definition 2.7, Theorem 2.2):

$$P_3^{27} = \frac{27!}{(27-3)!} = 17550$$

(b) A working group of equal standing is a **subset**, so this is a combination (Definition 2.8, Theorem 2.4):

$$\binom{27}{3} = \frac{27!}{3!\,(27-3)!} = 2925$$

(c) $17550/2925 = 6 = 3!$.

This is Theorem 2.4 read backwards. P_r^n counts each subset once for every one of the $r!$ orders in which its members could have been drawn, so dividing by $r!$ collapses those repetitions into one subset.

(d) $\binom{14}{3} = 364$.

The test to apply every time: name two of the chosen people and swap them. If the outcome has changed, order matters and you want P_r^n; if it has not, you want $\binom{n}{r}$.

Problem 6

seed 20260919

A risk committee of 3 members is to be formed from 20 eligible staff. All members serve on equal terms, so a committee is an unordered subset.

(a) How many committees are possible? ______________

(b) How many of them include the chief risk officer, who is one of the 20? ______________

(c) How many exclude her? ______________

(d) If the committee is drawn at random, all committees equally likely, what is the probability that she is on it? Give at least four significant figures. ______________

(e) Once the committee is seated, 3 of its 3 members form a drafting subcommittee, again unordered. How many subcommittees can be formed from a given committee? ______________

Solution

(a) Order does not matter, so this is a combination: $C_3^{20} = 1140$.

(b) Seat her first; the remaining $3-1$ seats are filled from the other $20-1$ staff, in $C_{3-1}^{20-1} = 171$ ways.

(c) Remove her from the pool entirely: $C_3^{20-1} = 969$.

The two counts add to the answer in (a), because every committee either contains her or does not.

(d) $171/1140 = 0.15$, which equals $3/20$ — her share of the seats.

(e) An unordered choice within the committee: $C_3^3 = 1$.

Problem 7

seed 20260919

A working party is to be drawn from 12 actuaries and 6 lawyers. Members serve on equal terms, so a working party is an unordered subset.

(a) In how many ways can 3 of the actuaries be chosen? ______________

(b) In how many ways can a party of 3 actuaries and 3 lawyers be formed? ______________

(c) Ignoring profession, how many parties of 6 people can be formed from the 18 candidates? ______________

(d) A party of 6 is drawn at random from all those in (c), equally likely. What is the probability that it contains exactly 3 actuaries? Give at least four significant figures. ______________

Solution

(a) $C_3^{12} = 220$.

(b) Choosing the actuaries and choosing the lawyers are two stages of one selection, so the mn rule multiplies the counts:

$$C_3^{12} \times C_3^{6} = 220 \times 20 = 4400$$

(c) $C_6^{18} = 18564$.

(d) The parties in (c) are the equally likely sample points and those in (b) are the favourable ones, so the probability is $4400/18564 = 0.237017884076708$.

Counting the favourable points and the total points with the same tool is what keeps the ratio honest: both are counts of unordered parties.

Problem 8

seed 20260919

A consultant's shelf holds 12 domestic funds and 5 international funds. 5 of the 17 funds are chosen at random, without replacement, every set of 5 equally likely. A sample point is the unordered set chosen.

(a) How many sample points does S contain? ______________

Give the remaining answers to at least four significant figures.

(b) $P(\text{all 5 chosen funds are domestic}) =$ ______________

(c) $P(\text{all 5 come from the same one of the two groups}) =$ ______________

(d) $P(\text{the chosen funds are not all from one group}) =$ ______________

Solution

(a) $C_5^{17} = 6188$ unordered sets.

(b) The favourable sets are the 5-subsets of the 12 domestic funds, of which there are $C_5^{12} = 792$, so the probability is $792/6188 = 0.127989657401422$.

(c) “All domestic” and “all international” are mutually exclusive, so their counts add: $(792 + 1)/6188 = 0.128151260504202$.

(d) The complement of (c): $1 - 0.128151260504202 = 0.871848739495798$.

Counting the complement is almost always the shorter road here; enumerating the mixed sets directly would mean a separate count for each possible split.

Problem 9

seed 20260919

A branch drawer holds 29 loan files, of which 2 were misreported at origination. An auditor draws 4 files at random, without replacement, and a sample point is the unordered set of files drawn.

Give the probabilities to at least four significant figures.

(a) How many sample points does S contain? ______________

(b) $P(\text{exactly 2 of the drawn files are misreported}) =$ ______________

(c) $P(\text{at least one drawn file is misreported}) =$ ______________

Solution

(a) The draw is unordered and without replacement, so S has

$$N = \binom{29}{4} = 23751$$

equally likely sample points.

(b) Build a sample point in the event: choose which 2 of the 2 misreported files appear, and which $4 - 2$ of the $29 - 2$ sound ones fill the rest. The two choices are independent stages, so the mn rule multiplies them:

$$n_a = \binom{2}{2}\binom{29-2}{4-2} = 351$$

and since the sample points are equally likely, $P = n_a/N = 0.0147783251231527$.

(c) “At least one” is quicker as the complement of “none”:

$$P(\text{at least one}) = 1 - \frac{\binom{29-2}{4}}{\binom{29}{4}} = 0.261083743842365$$

Counting, not listing, is what makes this tractable: at 29 files the sample space has 23751 points, and no one is writing those down.

Problem 10

seed 20260919

A batch of 27 invoices contains 5 that are overdue. An auditor samples 6 invoices at random without replacement, every subset of that size equally likely.

(a) How many sample points does S contain? ______________

Give the remaining answers to at least four significant figures.

(b) $P(\text{none of the sampled invoices is overdue}) =$ ______________

(c) $P(\text{exactly one is overdue}) =$ ______________

(d) $P(\text{at least two are overdue}) =$ ______________

Solution

Write $k = 5$ for the overdue invoices and $27 - 5$ for the rest. A sample containing exactly j overdue invoices is built in two stages — choose the j overdue ones, then the $6 - j$ others — so the mn rule gives $C^{5}_{j}\, C^{27-5}_{6-j}$ such samples.

(a) $C^{27}_{6} = 296010$.

(b) $j = 0$: $74613/296010 = 0.2520624303233$.

(c) $j = 1$: $131670/296010 = 0.444816053511706$.

(d) “At least two” is the complement of “none or exactly one”, and those two are mutually exclusive:

$$1 - (0.2520624303233 + 0.444816053511706) = 0.303122$$

Two counts and a subtraction replace the three or four counts that adding up $j = 2, 3, \ldots$ would require.

Problem 11

seed 20260919

A desk logs one recommendation per trading day for 15 days. Over that stretch it issued 6 **buy**, 2 **hold** and 7 **sell** recommendations. A sample point is the sequence of 15 letters read in date order, for example B, H, S, ... Reports carrying the same recommendation are indistinguishable in the sequence.

(a) How many distinct sequences are possible? ______________

(b) How many of them begin with a **buy**? ______________

Give the remaining answers to at least four significant figures.

(c) One of the sequences in (a) is selected at random, all equally likely. $P(\text{it begins with a buy}) =$

____

(d) $P(\text{a particular sequence is the one selected}) =$ ______________

Solution

(a) Choosing a sequence is the same as partitioning the 15 dated positions into a group of 6 holding B, a group of 2 holding H and a group of 7 holding S. Theorem 2.3 counts these partitions:

$$N = \frac{15!}{6!\,2!\,7!} = 180180$$

(b) Spend a B on the first position and partition the remaining $15 - 1$ positions among $6 - 1$ B's, 2 H's and 7 S's:

$$\frac{(15-1)!}{(6-1)!\,2!\,7!} = 72072$$

(c) $72072/180180 = 0.4$, which simplifies to $6/15$ — the share of the days that carried a buy.

(d) $1/180180 = 5.55000555000555e - 06$.

Note that the sequences in (a) are **not** equally likely as descriptions of how the desk actually behaves; they are equally likely only under the assumption stated in (c) and (d), which is what makes the sample-point method applicable there.

Problem 12

seed 20260919

24 analysts are assigned at random to several named desks. Desk 1 has 5 places and desk 2 has 8 places; the rest are spread over the remaining desks. Every partition of the 24 analysts into the desks of these stated sizes is equally likely — the number of such partitions is the multinomial coefficient of Theorem 2.3.

3 of the analysts joined the firm together and are known as the graduate team. Give each probability to at least four significant figures.

(a) $P(\text{all 3 of the graduate team are placed on desk 1}) =$ ______________

(b) $P(\text{all 3 of them are placed on desk 2}) =$ ______________

(c) $P(\text{none of them is placed on desk 1}) =$ ______________

Solution

Because every partition is equally likely, the 3 places occupied by the graduate team are as likely to be any 3 of the 24 places as any other. Fill their places one at a time.

(a) The first of them lands on desk 1 with probability $5/24$; given that, the second has $5 - 1$ desk-1 places left out of $24 - 1$ places, and so on:

$$P = \frac{5}{24} \cdot \frac{5-1}{24-1} \cdots = 0.00494071146245059$$

Equivalently, in the form of Theorem 2.3: count the partitions in which the team's 3 members all sit on desk 1 and divide by the total number of partitions. The ratio telescopes to the product above.

(b) The same argument with desk 2's 8 places gives 0.0276679841897233.

(c) Now each of them must take one of the $24 - 5$ places away from desk 1:

$$P = \frac{24-5}{24} \cdot \frac{24-5-1}{24-1} \cdots = 0.478754940711462$$

Note that (a) and (c) are not complements of one another: between them lies the event that the team is split, which is usually the likeliest outcome of the three. This is the calculation a mediation panel would want if someone alleged that an assignment announced as random was not.

Problem 13

seed 20260919

15 inspectors are to be split among three named shifts: 6 on the morning shift, 3 on the afternoon shift and 6 on the night shift. The shifts are distinguishable; the inspectors within a shift are not ranked.

(a) In how many ways can the morning shift alone be filled? ________________

(b) In how many ways can the full split be made? ________________

(c) One split is chosen at random, all splits equally likely. What is the probability that a particular inspector is placed on the morning shift? Give at least four significant figures. ________________

(d) A second plant splits its 12 inspectors into a day panel of 5 and a night panel holding the rest. In how many ways can that split be made? ________________

Solution

(a) An unordered choice of 6 from 15: $C_6^{15} = 5005$.

(b) Filling the three shifts in turn and applying the mn rule gives $C_6^{15} C_3^{15-6} C_6^{6}$, which is Theorem 2.3:

$$N = \frac{15!}{6!\,3!\,6!} = 420420$$

(c) By symmetry no inspector is favoured, and the morning shift holds 6 of the 15 places, so the probability is $6/15 = 0.4$.

The shifts being **named** is what makes this a partition count rather than a combination count: swapping the morning and afternoon rosters produces a different split.

(d) With only two named groups the partition count collapses to a single combination, since naming one group determines the other: $C_5^{12} = 792$.

Problem 14

seed 20260919

3 of 18 candidates are chosen at random for a review team, every team of that size equally likely. Two of the candidates, call them Aygun and Rashad, have worked together before.

(a) How many teams are possible? ______________

Give the remaining answers to at least four significant figures.

(b) $P(\text{both Aygun and Rashad are on the team}) =$ ______________

(c) $P(\text{neither is on the team}) =$ ______________

(d) $P(\text{exactly one of the two is on the team}) =$ ______________

(e) A second round draws a team of 4 from 17 candidates. How many teams are possible there? ______________

Solution

(a) $C_3^{18} = 816$.

(b) Place both, then fill the remaining $3-2$ seats from the other $18-2$ candidates: $C_{3-2}^{18-2} = 16$ teams, so the probability is 0.0196078431372549.

(c) Remove both from the pool: $C_3^{18-2} = 560$ teams, probability 0.686274509803922.

(d) Every team contains both, neither, or exactly one of the two, and the three events are mutually exclusive with union S. Hence $1 - 0.0196078431372549 - 0.686274509803922 = 0.294117647058823$.

(e) $C_4^{17} = 2380$.

Problem 15

seed 20260919

A reference number consists of one letter chosen from 16 permitted letters followed by 2 digits, each digit from 0 to 9 and repeats allowed.

(a) How many reference numbers are there? ______________

(b) 4 of the letters are reserved for internal use and may not appear on a client reference. How many client references are there? ______________

(c) Returning to all 16 letters: how many reference numbers have a first digit other than 0? ______________

(d) A reference number is drawn at random from those in (a), all equally likely. What is the probability it is a client reference? Give at least four significant figures. ______________

Solution

(a) The construction has $1+2$ stages, with 16 options at the first and 10 at each of the rest: $16 \times 10^2 = 1600$.

(b) Only the first stage changes, from 16 options to $16-4$: $(16-4) \times 10^2 = 1200$.

(c) The first digit now has 9 options and the remaining $2-1$ digits have 10 each: $16 \times 9 \times 10^{2-1} = 1440$.

(d) $1200/1600 = 0.75$, which is just $(16-4)/16$ — the digits cancel, since the restriction touches only the letter.

Problem 16

seed 20260919

A loan agreement offers 17 optional covenants. Any collection of them may be written into the contract, including all of them and none of them. A sample point is the set of covenants included.

(a) How many different contracts can be written? ______________

(b) How many contain exactly 2 covenants? ______________

(c) How many contain exactly 5 covenants? ______________

(d) How many contain at least one covenant? ______________

(e) A contract is drawn at random from those in (a), all equally likely. What is the probability that it contains exactly 2 covenants? Give at least four significant figures. ______________

Solution

(a) Each covenant is a two-option stage — in or out — and there are 17 of them, so the mn rule gives $2^{17} = 131072$.

(b) Contracts of a stated size are unordered selections: $C_2^{17} = 136$.

(c) $C_5^{17} = 6188$.

(d) Only the empty contract is excluded: $131072 - 1 = 131071$.

(e) $136/131072 = 0.00103759765625$.

Parts (a) and (b) sit either side of the identity $\sum_{r=0}^{n} C_r^n = 2^n$: counting the contracts by size and adding must give the same total as counting them one covenant at a time.

Problem 17

seed 20260919

9 agenda items are to be placed in order at a board meeting. 2 of them concern the same matter and the chair would like them taken one after another.

(a) How many orders of the 9 items are there? ______________

(b) How many orders place those 2 items in consecutive positions? ______________

(c) How many orders break them up? ______________

(d) An order is drawn at random from those in (a), all equally likely. What is the probability that the 2 items are consecutive? Give at least four significant figures. ______________

(e) At the following meeting there are 12 items and no adjacency requirement. In how many orders does one particular item come first? ______________

Solution

(a) $9! = 362880$.

(b) Glue the 2 related items into a single block. There are then $9 - 2 + 1$ objects to order, and the block itself can be read in $2!$ internal orders. The mn rule multiplies the two:

$$2! \times (9 - 2 + 1)! = 80640$$

(c) The complement within (a): $362880 - 80640 = 282240$.

(d) $80640/362880 = 0.2222222222222222$.

(e) Fix the chosen item in the first position and order the other $12 - 1$ freely: $(12 - 1)! = 39916800$.

Treating a constrained group as one object is the standard device: it turns a restriction into an ordinary mn-rule count.

Problem 18

seed 20260919

A portfolio holds 7 A-rated issues, 6 B-rated issues and 8 C-rated issues. 6 of the 21 issues are selected at random for review, without replacement, every set of 6 equally likely.

(a) How many sample points does S contain? ______________

Give the remaining answers to at least four significant figures.

(b) $P(\text{exactly 2 A-rated, 2 B-rated and 2 C-rated are selected}) =$ ______________

(c) $P(\text{no C-rated issue is selected}) =$ ______________

(d) $P(\text{at least one C-rated issue is selected}) =$ ______________

Solution

(a) $C_6^{21} = 54264$.

(b) Selecting from each rating is a separate stage, so the counts multiply:

$$C_2^7 C_2^6 C_2^8 = 8820$$

and the probability is $8820/54264 = 0.162538699690402$.

(c) Draw the whole review set from the $7 + 6$ non-C issues: $C_6^{7+6} = 1716$, probability 0.0316231755860239.

(d) The complement of (c): $1 - 0.0316231755860239 = 0.968376824413976$.

Problem 19

seed 20260919

A treasury desk holds 20 bonds. Each matures on one of 250 possible settlement dates, every date equally likely and the bonds' dates assigned independently. A sample point is an ordered 20-tuple of dates, so S contains 250^{20} equally likely points.

Give each probability to at least four significant figures.

(a) $P(\textbf{no two bonds mature on the same date}) =$ ______________

(b) $P(\textbf{at least two bonds mature on the same date}) =$ ______________

Solution

(a) Count the tuples with no repeated date by filling the 20 positions in turn. The first bond's date can be any of 250; the second any of the $250 - 1$ not already used; the third any of $250 - 2$, and so on. By the extension of the mn rule,

$$n_a = 250 \times (250 - 1) \times \cdots \times (250 - 20 + 1) = P_{20}^{250}$$

so, dividing by $N = 250^{20}$,

$$P = \frac{P_{20}^{250}}{250^{20}} = 0.458145653352486$$

(b) The complement: $1 - 0.458145653352486 = 0.541854346647514$.

Most people put the answer to (b) far too low. With 365 dates the probability of a clash passes one half at only 23 bonds, because the number of **pairs** that could clash grows like m^2 while the number of dates stays fixed — there are $\binom{20}{2}$ pairs here, not 20. It is the standard counter-example to probabilistic intuition, and a real scheduling problem for a desk that has to fund each maturity.

Problem 20

seed 20260919

A procurement office receives 12 bids. 4 of them will be shortlisted, and the shortlist is unordered.

(a) How many shortlists are possible? ______________

(b) From a given shortlist, the office will award first, second and third place, all to different bidders. In how many ways can the three places be awarded? ______________

(c) Separately, 8 reviewers are split into a senior panel of 5 and a junior panel of 3. The panels are named; the reviewers within a panel are not ranked. In how many ways can the split be made? ________________

(d) The shortlist is drawn at random from the lists in (a), all equally likely. Two of the 12 bids came from the same parent company. What is the probability that both are shortlisted? Give at least four significant figures.

____

Solution

(a) Unordered selection: $C_4^{12} = 495$.

(b) The three places are distinct, so order matters within the shortlist: $P_3^4 = 24$.

(c) Theorem 2.3 with two named groups:

$$\frac{8!}{5!\,3!} = 56$$

which is the same number as C_5^8 — with two groups, naming one determines the other.

(d) Put both related bids on the list and fill the remaining $4 - 2$ places from the other $12 - 2$ bids. The probability is

$$\frac{C_{4-2}^{12-2}}{C_4^{12}} = 0.0909090909090909$$

Each part uses a different tool, and the choice is settled by one question in each case: does order matter, and are the groups named?

Answer key

Problem	Answers
1	(a) 52, (b) 520, (c) 0.00192308
2	(a) 6859, (b) 5814, (c) 0.847645, (d) 6859
3	(a) 3.6288E+06, (b) 720, (c) 0.1, (d) 720
4	(a) 648, (b) 108, (c) 0.00154321
5	(a) 17550, (b) 2925, (c) 6, (d) 364
6	(a) 1140, (b) 171, (c) 969, (d) 0.15, (e) 1
7	(a) 220, (b) 4400, (c) 18564, (d) 0.237018
8	(a) 6188, (b) 0.12799, (c) 0.128151, (d) 0.871849
9	(a) 23751, (b) 0.0147783, (c) 0.261084
10	(a) 296010, (b) 0.252062, (c) 0.444816, (d) 0.303122
11	(a) 180180, (b) 72072, (c) 0.4, (d) 5.55001E-06
12	(a) 0.00494071, (b) 0.027668, (c) 0.478755

13	(a) 5005, (b) 420420, (c) 0.4, (d) 792
14	(a) 816, (b) 0.0196078, (c) 0.686275, (d) 0.294118, (e) 2380
15	(a) 1600, (b) 1200, (c) 1440, (d) 0.75
16	(a) 131072, (b) 136, (c) 6188, (d) 131071, (e) 0.0010376
17	(a) 362880, (b) 80640, (c) 282240, (d) 0.222222, (e) 3.99168E+07
18	(a) 54264, (b) 0.162539, (c) 0.0316232, (d) 0.968377
19	(a) 0.458146, (b) 0.541854
20	(a) 495, (b) 24, (c) 56, (d) 0.0909091