

Problem Set — Week 3, Session 1

Mathematical Statistics I · Wackerly §2.7 · ADA University

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260923

A branch classifies the 282 loan applications it received last quarter by whether the loan was secured and by whether it was approved.

	Secured	Unsecured
Approved:	138	63
Refused:	38	43

One of the 282 applications is drawn at random, so that every application is equally likely. Let A be the event that it was approved and S the event that it was secured.

Give every probability to at least four significant figures.

(a) $P(A \cap S) =$ _____

(b) $P(A | S) =$ _____

(c) $P(S | A) =$ _____

Problem 2

seed 20260923

Let A be the event that a mid-sized exporter raises new bank debt this year and B the event that it opens a new foreign subsidiary this year. For the firms on a regulator's register,

$$P(A) = 0.69, \quad P(B) = 0.50, \quad P(A \cap B) = 0.4450.$$

Give every probability to at least four significant figures.

(a) $P(A | B) =$ _____

(b) $P(B | A) =$ _____

(c) $P(A \cup B) =$ _____

(d) Are A and B independent, dependent, or mutually exclusive?

- ☐ Dependent
- ☐ Independent
- ☐ Mutually exclusive

Problem 3

seed 20260923

An insurer classifies its 270 policies in force by line of business and by whether a claim was made during the year.

	Claim	No claim	Total
Motor:	88	70	158
Household:	39	73	112
Total:	127	143	270

One policy is drawn at random. Let C be the event that a claim was made and M the event that the policy is a motor policy. Give every answer to at least four significant figures.

(a) $P(C) =$ _____

(b) $P(C \mid M) =$ _____

(c) C and M are not independent. How many motor policies would have had to make a claim — that is, what would the top-left cell have to be — for C and M to be independent? (A non-integer answer is expected; do not round it.) _____

Problem 4

seed 20260923

A fund holds 68 corporate bonds, of which 42 are investment grade and 26 are high yield. Two of them are selected at random for an audit, one after the other and **without replacement**. Let G_1 be the event that the first bond drawn is investment grade and G_2 the event that the second one is.

Give every probability to at least four significant figures.

(a) $P(G_1) =$ _____

(b) $P(G_2 \mid G_1) =$ _____

(c) $P(G_1 \cap G_2) =$ _____

(d) $P(G_2 \mid \bar{G}_1) =$ _____

Problem 5

seed 20260923

In a microfinance portfolio, B is the event that a borrower is taking a loan for the first time and A is the event that the borrower falls into arrears during the first year. The lender's records give

$$P(B) = 0.55, \quad P(A \mid B) = 0.42, \quad P(A) = 0.3600.$$

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A \cup B) =$ _____

(c) $P(B | A) =$ _____

Problem 6

seed 20260923

Two trading desks close their books independently. Let A be the event that the Baku desk reports a profit on a given day and B the event that the Istanbul desk does, with

$$P(A) = 0.74, \quad P(B) = 0.50,$$

and A and B independent. Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(\bar{A} \cap B) =$ _____

(c) $P(\bar{A} \cap \bar{B}) =$ _____

(d) $P(A | \bar{B}) =$ _____

Problem 7

seed 20260923

A rating agency will put a bank on review with one of exactly three outcomes at the next committee: an upgrade (A), a downgrade (B), or no change (C). Exactly one of the three occurs, and

$$P(A) = 0.44, \quad P(B) = 0.30.$$

Give every probability to at least four significant figures.

(a) $P(C) =$ _____

(b) $P(A \cup B) =$ _____

(c) $P(A | B) =$ _____

(d) Are A and B independent?

- ☐ Dependent
- ☐ Independent
- ☐ Not enough information

(e) $P(A | \bar{C}) =$ _____

Problem 8

seed 20260923

A data centre reaches the internet over two links that fail independently of one another. On a given day the primary link is available with probability **0.965** and the backup link with probability **0.855**.

Let A_1 and A_2 be the events that the primary and the backup link are available. Give every probability to at least four significant figures.

(a) $P(A_1 \cap A_2) =$ _____

(b) $P(\bar{A}_1 \cap \bar{A}_2) =$ _____

(c) $P(\text{at least one link is available}) =$ _____

(d) $P(\text{exactly one link is available}) =$ _____

Problem 9

seed 20260923

For borrowers in a distressed portfolio, let B be the event that a borrower misses a scheduled payment during the year and A the event that the borrower defaults outright. Every default begins with a missed payment, so $A \subset B$. The portfolio has

$$P(A) = 0.2368, \quad P(B) = 0.74.$$

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A \mid B) =$ _____

(c) $P(B \mid A) =$ _____

(d) $P(A \cup B) =$ _____

(e) $P(\bar{A} \cap B) =$ _____

Problem 10

seed 20260923

Two analysts call the direction of the manat-dollar rate for the coming quarter. They work from different data and their calls are independent. Analyst A is right with probability **0.87** and analyst B with probability **0.68**. Write A and B for the events that each is right.

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(\text{at least one of them is right}) =$ _____

(c) $P(A \mid \text{at least one is right}) =$ _____

(d) $P(\text{both are right} \mid \text{at least one is right}) =$ _____

Problem 11

seed 20260923

A bank reviews every card account opened last year. Branch A opened 890 accounts, of which 71 were later flagged for fraud; branch B opened 570 accounts, of which 29 were flagged. One account is drawn at random from all 1460 of them. Let F be the event that it was flagged and A the event that it was opened at branch A.

Give every probability to at least four significant figures.

(a) $P(F) =$ _____

(b) $P(F | A) =$ _____

(c) $P(A | F) =$ _____

Problem 12

seed 20260923

A reviewer with no real preference ranks 9 funds from first to last, assigning every one of the $9!$ possible orderings the same probability. Two of the funds are Javid and Gobustan. Define

A : Javid is ranked above Gobustan.

B : Javid is ranked first.

C : Javid is ranked 6th.

Give every probability to at least four significant figures.

(a) $P(A) =$ _____

(b) $P(B) =$ _____

(c) $P(A \cap B) =$ _____

(d) $P(A | C) =$ _____

(e) Are A and C independent?

☐ Independent

☐ Dependent

Problem 13

seed 20260923

A utility records whether each of its 462 business customers paid its last invoice late, by the region the customer is in.

	Late	On time	Total
Baku:	69	134	203
Ganja:	36	87	123
Sumqayit:	27	109	136

One customer is drawn at random. Let L be the event that it paid late and R_1, R_2, R_3 the events that it is in Baku, Ganja and Sumqayit.

Give every probability to at least four significant figures.

(a) $P(L) =$ _____

(b) $P(L | R_2) =$ _____

(c) $P(R_1 | L) =$ _____

(d) $P(R_1 \cup R_2 | L) =$ _____

Problem 14

seed 20260923

Two unrelated pieces of machinery in a plant break down independently in any given month. Let A be the event that the press breaks down and B that the furnace does, with $P(A) = 0.45$ and $P(B) = 0.29$.

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A \cup B) =$ _____

(c) A maintenance call-out is logged in any month in which at least one of the two breaks down. Given that a call-out was logged, what is the probability that the press broke down? _____

(d) Given that a call-out was logged, what is the probability that the press broke down and the furnace did not? _____

Problem 15

seed 20260923

An auditor pulls three invoices at random, one after another and without replacement, from a batch of 61 invoices of which 46 are sound and 15 contain an error. Let S_1, S_2, S_3 be the events that the first, second and third invoice drawn is sound.

Give every probability to at least four significant figures.

(a) $P(S_2 | S_1) =$ _____

(b) $P(S_3 | S_1 \cap S_2) =$ _____

(c) $P(S_1 \cap S_2 \cap S_3) =$ _____

(d) $P(\text{at least one of the three contains an error}) =$ _____

Problem 16

seed 20260923

Let B be the event that a registered firm is family owned and A the event that it has adopted electronic invoicing. A survey finds

$$P(B) = 0.69, \quad P(A | B) = 0.45, \quad P(A | \bar{B}) = 0.45.$$

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A) =$ _____

(c) $P(B | A) =$ _____

(d) $P(A \cup B) =$ _____

(e) Are A and B independent?

☐ Independent

☐ Dependent

Problem 17

seed 20260923

A bank holds loans to three unrelated borrowers whose defaults over the coming year are independent events D_1, D_2, D_3 with

$$P(D_1) = 0.40, \quad P(D_2) = 0.24, \quad P(D_3) = 0.19.$$

Give every probability to at least four significant figures.

(a) $P(\text{no borrower defaults}) =$ _____

(b) $P(\text{all three default}) =$ _____

(c) $P(\text{exactly one defaults}) =$ _____

(d) Given that exactly one borrower defaulted, what is the probability that it was borrower 1? _____

seed 20260923

Problem 18

On a register of firms, A is the event that a firm exports and B the event that it holds a bank loan, with

$$P(A) = 0.65, \quad P(B) = 0.47, \quad P(A \cap B) = 0.2255.$$

Give every answer to at least four significant figures.

(a) What would $P(A \cap B)$ have to be for A and B to be independent? _____

(b) $P(A | B) =$ _____

(c) What would $P(A | B)$ have to be for A and B to be independent? _____

seed 20260923

Problem 19

A plant draws power through two generators feeding a single switchgear panel. The plant runs if **at least one** generator is working **and** the switchgear is working. The three components work independently, with probabilities **0.93**, **0.80** and **0.94** respectively. Write G_1, G_2, W for the events that each works, and R for the event that the plant runs.

Give every probability to at least four significant figures.

(a) $P(G_1 \cup G_2) =$ _____

(b) $P(R) =$ _____

(c) $P(G_1 | R) =$ _____

