

Problem Set — Week 3, Session 2

Mathematical Statistics I · Wackerly §2.8 · ADA University

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260926

A retail bank's customers are surveyed. Let A be the event that a customer uses the mobile app and B the event that the customer holds a credit card:

$$P(A) = 0.42, \quad P(B) = 0.45, \quad P(A \cap B) = 0.32.$$

Give every probability to at least four significant figures.

(a) $P(A \cup B) =$ _____

(b) $P(\bar{A} \cap \bar{B}) =$ _____

(c) $P(A \cap \bar{B}) =$ _____

Problem 2

seed 20260926

A construction firm bids for public tenders. Let A be the event that a bid is filed before the deadline and B the event that the bid wins. Historically

$$P(A) = 0.43, \quad P(B | A) = 0.57, \quad P(B) = 0.4900.$$

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A \cup B) =$ _____

(c) $P(A | B) =$ _____

Problem 3

seed 20260926

An operations manual records that a particular machine injures an operator on about 1 shift in 42. Shifts are independent of one another.

Give every probability to at least four significant figures, and enter them as decimals.

(a) $P(\text{an injury on a single shift}) =$ _____

(b) $P(\text{at least one injury in 8 shifts}) =$ _____

(c) A colleague argues that since the chance is 1 in 42 per shift, an operator working 42 shifts is certain to be injured. Compute the correct probability of at least one injury in 42 shifts. _____

Problem 4

seed 20260926

Two events are defined on the firms in a supervisory register: A , the firm filed its accounts on time, and B , the firm has no overdue tax. It is known that

$$P(A) = 0.71 \quad \text{and} \quad P(B) = 0.75,$$

and nothing else. Give every probability to at least four significant figures.

(a) What is the smallest value $P(A \cap B)$ could take? _____

(b) What is the largest value $P(A \cap B)$ could take? _____

(c) Could $P(A \cap B) = 0.77$?

- ☐ Impossible: too small
- ☐ Possible
- ☐ Impossible: too large

Problem 5

seed 20260926

An invoice is settled early (E), on its due date (D), late (L), or not at all (N). Exactly one of the four happens, and

$$P(E) = 0.1900, \quad P(D) = 0.4000, \quad P(L) = 0.2300.$$

Give every probability to at least four significant figures.

(a) $P(N) =$ _____

(b) $P(E \cup D) =$ _____

(c) $P(\bar{L}) =$ _____

(d) $P(D \cup L) =$ _____

Problem 6

seed 20260926

A telecommunications regulator receives complaints about three faults: slow speed (A), dropped connections (B) and billing errors (C). For a subscriber drawn at random from the complaints register,

$$P(A) = 0.4000, \quad P(B) = 0.3500, \quad P(C) = 0.4100,$$

$$P(A \cap B) = 0.1300, \quad P(A \cap C) = 0.1400, \quad P(B \cap C) = 0.1300, \quad P(A \cap B \cap C) = 0.0500.$$

Give every probability to at least four significant figures.

(a) $P(A \cup B \cup C) =$ _____

(b) $P(\text{none of the three faults}) =$ _____

(c) $P(\text{slow speed and neither of the other two}) =$ _____

Problem 7

seed 20260926

A tender passes through three stages in order: a technical screen (T), a financial screen (F), and the award decision (W). A bid reaches a stage only if it has passed the previous one. For a bid drawn at random,

$$P(T) = 0.66, \quad P(F | T) = 0.65, \quad P(W | T \cap F) = 0.71.$$

Give every probability to at least four significant figures.

(a) $P(T \cap F) =$ _____

(b) $P(T \cap F \cap W) =$ _____

(c) $P(\text{the bid is not awarded}) =$ _____

(d) $P(\text{the bid passes the technical screen but fails the financial one}) =$ _____

Problem 8

seed 20260926

Three risks are monitored at a plant: a power interruption (A_1), a raw material shortage (A_2) and a labour stoppage (A_3). A shortage and a stoppage never occur in the same month, so $P(A_2 \cap A_3) = 0$, while

$$P(A_1) = 0.3200, \quad P(A_2) = 0.26, \quad P(A_3) = 0.27,$$

$$P(A_1 \cap A_2) = P(A_1 \cap A_3) = 0.06.$$

Give every probability to at least four significant figures.

(a) $P(A_1 \cap A_2 \cap A_3) =$ _____

(b) $P(\text{at least one of the three risks occurs}) =$ _____

(c) $P(\text{none of the three occurs}) =$ _____

(d) By how much does $P(A_1) + P(A_2) + P(A_3)$ exceed the answer to (b)? _____

Problem 9

seed 20260926

A survey of households reports that **0.37** subscribe to fixed broadband (A), **0.40** subscribe to pay television (B), and **0.6920** subscribe to at least one of the two.

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(A | B) =$ _____

(c) Are A and B independent?

☐ Independent

☐ Dependent

(d) If instead the two subscriptions were independent, what would $P(A \cup B)$ be? _____

seed 20260926

Problem 10

In a sample of manufacturing firms, A is the event that a firm exports to Turkey and B the event that it exports to Russia, with

$$P(A) = 0.32, \quad P(B) = 0.36, \quad P(A \cap B) = 0.24.$$

Give every probability to at least four significant figures.

(a) $P(\overline{A \cup B}) =$ _____

(b) $P(\text{the firm exports to exactly one of the two}) =$ _____

(c) $P(\bar{A} \cap B) =$ _____

seed 20260926

Problem 11

Three suppliers deliver to a plant. Their deliveries are late independently of one another, with probabilities

$$P(L_1) = 0.18, \quad P(L_2) = 0.23, \quad P(L_3) = 0.26.$$

Give every probability to at least four significant figures.

(a) $P(L_1 \cap L_2) =$ _____

(b) $P(L_1 \cup L_2) =$ _____

(c) $P(L_1 \cup L_2 \cup L_3) =$ _____

(d) $P(L_1 \cap L_2 \cap L_3) =$ _____

seed 20260926

Problem 12

For two events defined on a portfolio it is known only that

$$P(A) = 0.27 \quad \text{and} \quad P(B) = 0.30.$$

Give every probability to at least four significant figures.

(a) The smallest possible value of $P(A \cap B)$ is _____

(b) The largest possible value of $P(A \cap B)$ is _____

(c) The smallest possible value of $P(A \cup B)$ is _____

(d) The largest possible value of $P(A \cup B)$ is _____

Problem 13

seed 20260926

Two inspectors examine the same consignment independently of each other's paperwork, but they are looking at the same goods, so their verdicts are not independent events. Let A and B be the events that inspector 1 and inspector 2 respectively flag the consignment. It is known that

$$P(A) = 0.23, \quad P(B | A) = 0.67, \quad P(B | \bar{A}) = 0.21.$$

Give every probability to at least four significant figures.

(a) $P(A \cap B) =$ _____

(b) $P(B) =$ _____

(c) $P(A \cup B) =$ _____

(d) $P(\text{neither inspector flags it}) =$ _____

Problem 14

seed 20260926

A municipal survey of small shops finds that **0.39** accept card payments (A), **0.7600** accept cards or issue electronic receipts or both ($A \cup B$), and **0.16** do both ($A \cap B$).

Give every probability to at least four significant figures.

(a) $P(B) =$ _____

(b) $P(\text{the shop does neither}) =$ _____

(c) $P(B | A) =$ _____

(d) $P(A \cap \bar{B}) =$ _____

Problem 15

seed 20260926

A batch of 35 castings contains 7 that are defective. An inspector draws 5 of them at random without replacement.

Give every probability to at least four significant figures.

(a) $P(\text{none of the 5 drawn is defective}) =$ _____

(b) $P(\text{at least one of the 5 drawn is defective}) =$ _____

(c) Suppose instead each casting were returned to the batch before the next draw. What would the probability of at least one defective be then? _____

Problem 16

seed 20260926

An exploration company drills wells that strike gas independently of one another, each with probability **0.120**. The board wants the probability of at least one strike to be at least **0.85**.

(a) What is the smallest number of wells that achieves this? Enter a whole number. _____

Give the remaining probabilities to at least four significant figures.

(b) With that many wells, $P(\text{at least one strike}) =$ _____

(c) With one well fewer, $P(\text{at least one strike}) =$ _____

Problem 17

seed 20260926

A bank's operational risk register lists three independent failure events for the coming quarter — a payments outage, a reconciliation error and a third-party breach — with probabilities

0.13, 0.16, 0.18.

Give every probability to at least four significant figures.

(a) The register reports “total operational risk” by adding the three figures. What does it report? _____

(b) What is the correct probability that at least one of the three occurs? _____

(c) By how much does the reported figure overstate the correct one? _____

Problem 18

seed 20260926

A bank holds loans to two firms in the same supply chain. Over the coming year firm 1 defaults with probability **0.07** and firm 2 with probability **0.08**. The two are not independent: if firm 1 defaults, firm 2's default probability is **4.5** times its unconditional value.

Give every probability to at least four significant figures.

(a) $P(D_2 \mid D_1) =$ _____

(b) $P(D_1 \cap D_2) =$ _____

(c) $P(D_1 \cup D_2) =$ _____

(d) $P(\text{neither firm defaults}) =$ _____

Problem 19

seed 20260926

Among a telecoms operator's customers, let C be the event that a customer is on a fixed tariff, A that the customer complains about speed and B that the customer complains about billing. The operator reports

$P(C) = 0.42, \quad P(A \mid C) = 0.45, \quad P(B \mid C) = 0.49, \quad P(A \cap B \mid C) = 0.28.$

Give every probability to at least four significant figures.

(a) $P(A \cup B \mid C) =$ _____

(b) $P(\bar{A} \cap \bar{B} \mid C) =$ _____

(c) $P(A \cap B \cap C) =$ _____

