Week 4, Problem Set 1

Wackerly 2.9 - the event-composition method

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A retail bank sends a pre-approved credit-card offer to all of its current account holders. Of these customers, 52% are salaried employees and the remaining 48% are self-employed. Among salaried customers 56% accept the offer; among the self-employed 18% accept.

One customer is chosen at random. Let F be the event that the customer accepts the offer, S that the customer is salaried, and E that the customer is self-employed.

Give every probability to at least four significant figures.

(a) $P(F \cap S) =$ ____________

(b) $P(F \cap E) =$ ____________

(c) $P(F) =$ ____________

(d) The probability that the customer is self-employed and declines the offer: $P(E \cap \overline{F}) =$ ____________

Problem 2

seed 20260925

A payments company screens every card transaction with two fraud-detection engines bought from different vendors, each trained on its own data and running on its own servers, so that they operate independently. Each engine fails to flag a fraudulent transaction with probability 0.140.

A fraudulent transaction arrives.

Give every probability to at least four significant figures. You may enter an arithmetic expression such as 0.05^3 instead of a decimal.

(a) The probability that it goes undetected (no engine flags it): ____________

(b) The probability that it is flagged by both engines: ____________

(c) The probability that it is flagged by exactly one of the engines: ____________

(d) On a given day 90 fraudulent transactions arrive, and the engines treat each one independently of the others. What is the probability that at least one of the 90 goes undetected? ____________

Problem 3

seed 20260925

A salary payment from a company's account to an employee's account at another bank passes through four stages, in order, and completes only if every stage works:

1. the payer bank's online banking gateway accepts the instruction (probability 0.959);
2. the interbank messaging network delivers it (probability 0.995);
3. the central bank's real-time gross settlement system settles it (probability 0.969);
4. the beneficiary bank credits the employee's account (probability 0.993).

The stages are run by different institutions on separate infrastructure, and they succeed or fail independently of one another.

Give every probability to at least four significant figures.

(a) The probability that the payment completes: ______________

(b) The probability that the payment does not complete: ______________

(c) The probability that the payment gets through stages 1 and 2 and then fails at the settlement stage 3:

____

Problem 4

seed 20260925

A brokerage's trading screens take prices from a primary market-data feed. If the primary feed fails during a session, the system switches to a first backup feed; if that also fails, it switches to a second backup. The three feeds come from different vendors over physically separate lines, and during a session they fail independently with probabilities 0.035 (primary), 0.140 (first backup) and 0.11 (second backup).

Give every probability to at least four significant figures.

(a) The probability that all three feeds fail during a session: ______________

(b) The probability that the screens do not go dark, that is, that the system of three feeds does not completely fail:

____

(c) The probability that the second backup is called into service: ______________

(d) The probability that the session is served by the first backup (the primary fails and the first backup works):

____

Problem 5

seed 20260925

A microfinance lender's scoring system sends an incoming loan application to manual review with probability 0.055. Applications are scored one after another, and whether one is sent to review is independent of every other.

Watch the applications as they arrive on a given morning.

Give every probability to at least four significant figures.

(a) What is the probability that the 12th application is the first one sent to manual review? ________

(b) What is the probability that at least one of the first 11 applications is sent to manual review? ________

Problem 6

seed 20260925

A bank's trading desk reports a daily 97.5% Value-at-Risk figure. The model is correctly calibrated, so on any trading day the desk's loss exceeds the reported VaR (a "breach") with probability 0.025, and, as the standard backtest assumes, breaches on different days are independent.

Consider the next 24 trading days.

Give every probability to at least four significant figures. You may enter an arithmetic expression such as $0.97^{1}2$ instead of a decimal.

(a) The probability of no breach in the 24 days: ________

(b) The probability of at least one breach in the 24 days: ________

(c) The probability that each of the first three days is a breach: ________

Problem 7

seed 20260925

A bank's internal audit team selects two of the 18 corporate loan files in a branch's portfolio for a detailed review. The first file is drawn at random from all 18; the second is drawn at random from the 17 that remain. The six files with the largest exposures are of particular interest to the auditors; call the other 12 files "small".

Give every probability to at least four significant figures.

(a) One particular large-exposure file is known as file L. Find the probability that file L is drawn first and a small file is drawn second. ________

(b) Find the probability that exactly one of the six large-exposure files is selected. ________

(c) Find the probability that both selected files are large-exposure files. ________

(d) Find the probability that neither selected file is a large-exposure file. ________

(e) The same week, a different audit team, working independently, draws two of the 22 loan files at a second branch in the same way; that branch also has six large-exposure files. Find the probability that neither audit selects a large-exposure file. ________

Problem 8

seed 20260925

A compliance officer receives a batch of 14 supplier invoices, exactly two of which are forged. She checks the invoices one at a time, in a random order, and a check always reveals whether the invoice in hand is forged.

Give every probability to at least four significant figures.

(a) What is the probability that the second (last) forged invoice is found on the 7th check? ________

(b) What is the probability that both forged invoices are among the first 4 invoices checked? _______________

(c) Given that exactly one of the two forged invoices was found in the first two checks, what is the probability that the remaining forged invoice is found on the 3rd or the 4th check? _______________

Problem 9

seed 20260925

A treasury clerk must release a payment in the bank's corporate-banking portal and has been given a list of 24 one-time authorisation codes, only one of which is valid for this payment. Having no idea which one it is, the clerk picks a code at random and tries it; if it is rejected, he crosses it off and picks at random from the codes that remain, and so on until the valid code is found.

Give every probability to at least four significant figures.

(a) The probability that the valid code is found on the first try: _______________

(b) The probability that the valid code is found on the 3rd try: _______________

(c) The portal locks the payment if three wrong codes are entered before the valid one. What is the probability that the clerk releases the payment? _______________

(d) Suppose instead the clerk forgets which codes he has tried, so each try is a fresh random pick from all 24 codes (independent of earlier tries), and the portal still locks after three wrong codes. What is the probability that he releases the payment? _______________

Problem 10

seed 20260925

A bank's anti-money-laundering screen raises an alert on 83.5% of illicit transfers and, wrongly, on 19.0% of legitimate ones. An investigator is handed two transfers, T_1 and T_2, and it is known for certain that exactly one of them is illicit (but not which). The screen treats each transfer on its own, so its verdicts on the two are independent.

Give every probability to at least four significant figures.

(a) The probability that the screen raises an alert on both transfers: _______________

(b) The probability that it alerts on the illicit transfer and not on the legitimate one: _______________

(c) The probability that it is completely wrong: it alerts on the legitimate transfer and not on the illicit one.

____

(d) The probability that it alerts on at least one of the two: _______________

Problem 11

seed 20260925

A bank launches its new mobile-banking app with an advertising campaign on social media and on television. Market research on the target population finds that a randomly chosen adult sees the social-media ad with probability 0.13, sees the TV ad with probability 0.38, and sees both with probability 0.04. Of those who see at least one of the ads, a proportion 0.38 download the app within a month; of those who see neither ad, a proportion 0.06 download it anyway.

A randomly chosen adult is observed for a month. Let S and T be the events that the person sees the social-media ad and the TV ad, and D the event that the person downloads the app.

Give every probability to at least four significant figures.

(a) $P(S \cup T) =$ ______________

(b) The probability that the person sees neither ad: ______________

(c) $P(D) =$ ______________

(d) The probability that the person downloads the app without having seen either ad: ______________

Problem 12

seed 20260925

A bank has lent to two unrelated borrowers: a construction firm in Baku and an agricultural cooperative in the Ganja region. They operate in different sectors and regions and have no common owners, and the bank treats their defaults over the coming year as independent. Let D_1 be the event that the construction firm defaults, with $P(D_1) = 0.14$, and D_2 the event that the cooperative defaults, with $P(D_2) = 0.07$.

The risk committee asks for the probability of the event E: “exactly one of the two loans defaults”.

Consider the following compositions.

(i) $(D_1 \cap \overline{D_2}) \cup (\overline{D_1} \cap D_2)$

(ii) $D_1 \cup D_2$

(iii) $\overline{D_1} \cap \overline{D_2}$

(iv) $\overline{D_1 \cap D_2}$

(v) $D_1 \cap D_2$

(a) Which composition is exactly the event E (the same set of sample points)?

- ○ (i)
- ○ (ii)
- ○ (iii)
- ○ (iv)
- ○ (v)

(b) Find $P(E)$ to at least four significant figures. ______________

Problem 13

seed 20260925

A card payment at a shop in Baku is authorised through three electronic components A, B and C. The architecture is as follows.

An authorisation request must first pass the acquirer gateway A. After that, it is sent to the issuer's processor B; if B is down it is sent instead to B's hot-standby processor C, and either B or C can authorise it.

The three components are run by different operators in different data centres and work or fail independently, with probabilities of working $P(A) = 0.97$, $P(B) = 0.87$ and $P(C) = 0.96$ during a given transaction. (Here A, B, C also denote the events that the components work.) Let S be the event that the request is authorised.

Consider the following compositions.

(i) $S = A \cap B \cap C$

(ii) $S = A \cup B \cup C$

(iii) $S = A \cap (B \cup C)$

(iv) $S = (A \cap B) \cup C$

(v) $S = A \cup (B \cap C)$

Give every probability to at least four significant figures.

(a) Which composition describes this architecture?

- ○ (i)
- ○ (ii)
- ○ (iii)
- ○ (iv)
- ○ (v)

(b) $P(S) =$ ________________

(c) The probability that the request cannot be authorised: ________________

Problem 14

seed 20260925

At 90 minutes before the central bank's settlement cut-off, a bank's treasury desk discovers it is short of funds and must borrow overnight from another bank. There are many potential counterparties, and each one, independently of the others, has spare liquidity to lend with probability 0.46. The desk has one dealer, who can check only one counterparty at a time: a check takes 25 minutes and reveals whether that counterparty can lend. As soon as a lender is found, executing and settling the transfer takes a further 20 minutes, and the transfer must be complete by the cut-off.

(a) What is the largest number of counterparties the dealer can check so that, if the last of them turns out to be a lender, the transfer is still complete by the cut-off? ________________

(b) What is the probability that the bank borrows the funds in time? ________________

(c) What is the probability that the bank borrows in time **and** the lender is the last counterparty that could usefully be checked? ________________

Give probabilities to at least four significant figures.

Problem 15

seed 20260925

A Baku exporter of dried fruit has three shipments leaving this week to different buyers, each by a different route and carrier: one by rail through Georgia, one by Caspian ferry to Kazakhstan, and one by air freight to the Gulf. Each shipment is delayed beyond its contractual date with probability 0.09, 0.24 and 0.13 respectively, and because the routes, carriers and customs posts are unrelated, delays occur independently.

Give every probability to at least four significant figures.

(a) The probability that no shipment is delayed: ________________

(b) The probability that at least one shipment is delayed: ________________

(c) The probability that exactly one shipment is delayed: ________________

(d) The probability that two or more shipments are delayed: ________________

Problem 16

seed 20260925

The SME desk of a bank has two credit officers. For a standard working-capital loan from an established client, officer 1 approves every application on first submission; officer 2 sends every first submission back to the client for additional documents, which the desk records as a first-round rejection. The desk's workload system routes each application to officer 2 with probability 0.455 and to officer 1 otherwise, independently from one application to the next.

This month four established clients each submit one such application, on different days.

Give every probability to at least four significant figures.

(a) What is the probability that exactly three of the four applications are rejected in the first round?

____

(b) What is the probability that all four are approved on first submission? ________________

(c) What is the probability that at least one application is rejected in the first round? ________________

Problem 17

seed 20260925

A trader buys a stock and immediately places two orders: a take-profit order above the current price and a stop-loss order below it. The position is closed on the first day on which the price reaches either level. In a simple model of the stock, on each trading day the price reaches the take-profit level with probability 0.110, reaches the stop-loss level with probability 0.035, and otherwise reaches neither; it never reaches both on the same day, and what happens on different days is independent.

Give every probability to at least four significant figures.

(a) What is the probability that the position is closed at the take-profit level on day 4 (the 4th trading day)?

____

(b) What is the probability that the position is still open at the end of day 4? ________________

(c) What is the probability that the take-profit level is reached before the stop-loss level? ______________

(d) What is the probability that the position is closed at the take-profit level within the first 4 trading days?

____

Problem 18

seed 20260925

A Baku importer pays a supplier in Turkey. The payment leaves the importer's bank S, must reach the supplier's bank R, and in between can travel by either of two correspondent routes:

- **Route 1:** correspondent bank C_1, which passes it on through its own clearing bank H_1 (both must work);
- **Route 2:** correspondent bank C_2, which has a direct account at R.

The payment is sent by route 1; if route 1 fails at either C_1 or H_1, it is re-sent by route 2. It arrives if S works, at least one route works, and R works. On a given day the five institutions work independently, with probabilities

$$P(S) = 0.975,\ P(C_1) = 0.97,\ P(H_1) = 0.93,\ P(C_2) = 0.93,\ P(R) = 0.984,$$

where each letter also denotes the event that the institution works.

Give every probability to at least four significant figures.

(a) The probability that route 1 works: ______________

(b) The probability that at least one of the two routes works: ______________

(c) The probability that the payment arrives: ______________

(d) The probability that the payment does not arrive: ______________

(e) The probability that the payment arrives **only because** of the backup, that is, it arrives and route 1 has failed:

____

(f) Given that the payment arrived, the probability that it went by route 2: ______________

Problem 19

seed 20260925

An exchange's matching engine takes prices from independent market-data feeds, supplied by different vendors over separate lines. Each feed is working at a given moment with probability 0.825, independently of the others.

Give every probability to at least four significant figures.

(a) With three feeds, the engine uses a voting rule: it publishes a price only if at least two of the three feeds are working (so that they can be cross-checked). What is the probability that it can publish? ______________

(b) With the same three feeds, what would that probability be if the engine needed only one working feed?

____

(c) The regulator requires that, under the one-working-feed rule, the probability that at least one feed is working be at least 0.999999. What is the smallest number of feeds that achieves this? ______________

(d) Return to three feeds under the voting rule of (a). The engine also needs at least one of its two matching servers to be working; each server works with probability 0.93, independently of the other and of the feeds. What is the probability that the whole system can publish a price? ______________

Problem 20

seed 20260925

An external auditor is testing a company's file of 28 expense claims for the year. Unknown to the auditor, exactly 6 of the claims contain an error. The auditor draws claims at random, one at a time and without replacement (a claim already examined is not put back).

Give every probability to at least four significant figures.

(a) The auditor examines a sample of 5 claims. What is the probability that none of them contains an error?

____

(b) What is the probability that the sample of 5 contains at least one error claim? ______________

(c) Suppose instead the auditor keeps drawing until the first error claim is found. What is the probability that the first error claim is the third one examined? ______________

(d) A junior colleague approximates (b) by treating the 5 draws as if they were made with replacement, so that they are independent. What value does the colleague get? ______________

(e) What is the smallest sample size (drawing without replacement) for which the probability of finding at least one error claim is at least 0.90? ______________