

Week 4, Problem Set 2

Wackerly 2.10 - total probability and Bayes' rule

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A mobile operator's subscribers are either on a prepaid plan or on a postpaid contract, and nobody is on both. A subscriber is chosen at random. Let B be the event that the subscriber is prepaid and C the event that the subscriber leaves the operator (churns) next month. The operator's records give

$$P(B) = 0.60, \quad P(C | B) = 0.086, \quad P(C | \bar{B}) = 0.013.$$

Give every probability to at least four significant figures.

(a) $P(B \cap C) =$ _____

(b) $P(\bar{B} \cap C) =$ _____

(c) $P(C) =$ _____

Problem 2

seed 20260925

A Baku exporter of processed fruit invoices buyers in exactly one of three markets: the EU (B_1), the CIS (B_2) or Asia (B_3). An invoice is chosen at random from last year's book, and L is the event that it was paid late. Of all invoices, a fraction **0.34** went to the EU and **0.33** to the CIS; the rest went to Asia. The late-payment rates by market were

$$P(L | B_1) = 0.045, \quad P(L | B_2) = 0.13, \quad P(L | B_3) = 0.17.$$

Give every probability to at least four significant figures.

(a) $P(B_3) =$ _____

(b) $P(L) =$ _____

(c) The probability that a randomly chosen invoice was paid on time: _____

Problem 3

seed 20260925

A bank buys the card readers for its ATMs from two suppliers. A fraction **0.47** of its readers come from supplier 1 and the rest from supplier 2. Let B be the event that a reader is from supplier 1 and D the event that it fails its first-year inspection. From service records,

$$P(D | B) = 0.038 \quad \text{and} \quad P(D | \bar{B}) = 0.035.$$

A reader chosen at random fails its inspection. Give every probability to at least four significant figures.

(a) $P(D) =$ _____

(b) The probability that the failed reader came from supplier 1: $P(B | D) =$ _____

(c) The probability that it came from supplier 2: $P(\bar{B} | D) =$ _____

seed 20260925

Problem 4

A household survey in Baku asks whether the central bank should cut its policy rate. Let M be the event that a randomly chosen household has a mortgage and F the event that it favours a cut. The survey finds

$$P(M) = 0.20, \quad P(F | M) = 0.88, \quad P(F | \bar{M}) = 0.37.$$

Give every probability to at least four significant figures.

(a) $P(F) =$ _____

(b) A household that favours a cut is chosen at random. The probability that it has a mortgage, $P(M | F) =$ _____

(c) A household that does **not** favour a cut is chosen at random. The probability that it has a mortgage, $P(M | \bar{F}) =$ _____

seed 20260925

Problem 5

A loan officer has 30 consumer-loan applications on her desk this week: 15 from salaried workers and 15 from self-employed applicants. Each salaried application is approved with probability **0.78** and each self-employed application with probability **0.62**. After the decisions are made, one file is pulled at random from the 30 for a quality review.

Give every probability to at least four significant figures.

(a) The probability that the file pulled is a rejection: _____

(b) The file pulled is a rejection. The probability that it is from a self-employed applicant: _____

(c) If instead the file pulled had been an approval, the probability that it is from a self-employed applicant: _____

seed 20260925

Problem 6

A card issuer's real-time screen flags suspicious transactions. Let F be the event that a transaction is fraudulent and A the event that the screen flags it. The issuer knows

$$P(F) = 0.0020, \quad P(A | F) = 0.98, \quad P(A | \bar{F}) = 0.035.$$

Give every probability to at least four significant figures. Very small answers may be entered in scientific notation with a capital E, e.g. 2.5E-5 (a lowercase e is read as the constant e).

(a) The probability that a randomly chosen transaction is flagged, $P(A) =$ _____

(b) A transaction has just been flagged. The probability that it is fraudulent, $P(F | A) =$ _____

(c) A transaction was not flagged. The probability that it is nevertheless fraudulent, $P(F | \bar{A}) =$ _____

Problem 7

seed 20260925

A bank's corporate loan book is split across four internal rating grades, from grade 1 (safest) to grade 4 (riskiest). A loan is chosen at random; let B_i be the event that it is rated grade i and A the event that it defaults within a year.

	grade 1	grade 2	grade 3	grade 4
$P(B_i)$	0.34	0.25	0.21	0.20
$P(A B_i)$	0.003	0.019	0.049	0.124

Give every probability to at least four significant figures.

(a) The one-year default probability of a randomly chosen loan, $P(A) =$ _____

(b) A loan has defaulted. The probability that it was rated grade 4, $P(B_4 | A) =$ _____

(c) The probability that a defaulted loan was rated grade 1, $P(B_1 | A) =$ _____

(d) Which grade accounts for the largest share of this year's defaults?

- ☐ Grade 1
- ☐ Grade 2
- ☐ Grade 3
- ☐ Grade 4

Problem 8

seed 20260925

A consumer lender's scorecard rejects some applicants. Let G be the event that an applicant would repay (a "good" borrower) and R the event that the scorecard rejects the application. From a back-test on past loans,

$$P(\bar{G}) = 0.040, \quad P(R | \bar{G}) = 0.89, \quad P(R | G) = 0.17.$$

Give every probability to at least four significant figures.

(a) $P(R) =$ _____

(b) The probability that a rejected applicant is a bad borrower, $P(\bar{G} | R) =$ _____

(c) The probability that an accepted applicant is a bad borrower, $P(\bar{G} | \bar{R}) =$ _____

(d) The probability that a rejected applicant is a good borrower, $P(G | R) =$ _____

Problem 9

seed 20260925

A bank's anti-money-laundering (AML) system scans outgoing transfers. On historical evidence, 10 transfers in every 10,000 are part of a laundering scheme. Let L be the event that a randomly chosen transfer is one of them and A the event that the system raises an alert on it. The system alerts on a fraction **0.88** of laundering transfers

and on a fraction **0.011** of clean ones.

Give every probability to at least four significant figures. Very small answers may be entered in scientific notation with a capital E, e.g. 2.5E-5 (a lowercase e is read as the constant *e*).

(a) $P(A) =$ _____

(b) The probability that an alerted transfer really is laundering, $P(L | A) =$ _____

(c) The probability that a transfer with no alert is laundering, $P(L | \bar{A}) =$ _____

(d) The **lift** of the alert, $P(L | A)/P(L) =$ _____

Problem 10

seed 20260925

A pipeline operator lays steel pipe sections bought from three mills. Let B_i be the event that a randomly chosen section came from mill i and D the event that ultrasonic testing finds a weld defect in it.

	mill 1	mill 2	mill 3
$P(B_i)$	0.47	0.34	0.19
$P(D B_i)$	0.016	0.037	0.054

A section has just failed testing. Give every probability to at least four significant figures.

(a) $P(D) =$ _____

(b) $P(B_1 | D) =$ _____

(c) $P(B_3 | D) =$ _____

(d) Which mill is the most likely source of the defective section?

- ☐ Mill 1
☐ Mill 2
☐ Mill 3

Problem 11

seed 20260925

A motor insurer's underwriters sort policyholders into three risk classes, though the class of an individual driver is never observed directly. Let B_1, B_2, B_3 be the events that a randomly chosen policyholder is low, medium or high risk, and C the event that the policyholder makes at least one claim during the year.

	low	medium	high
$P(B_i)$	0.53	0.29	0.18
$P(C B_i)$	0.045	0.14	0.30

Give every probability to at least four significant figures.

(a) $P(C) =$ _____

(b) A policyholder claims this year. The probability that she is high risk, $P(B_3 | C) =$ _____

(c) A policyholder has a claim-free year. The probability that he is high risk, $P(B_3 | \bar{C}) =$ _____

(d) $P(B_1 | C) =$ _____

Problem 12

seed 20260925

A tax authority uses a risk model to select firms for audit. Let E be the event that a firm under-declares its VAT and A the event that the model selects it. A fraction **0.012** of firms under-declare. Following Exercise 2.126, call $P(A | E)$ the **sensitivity** of the model, $P(\bar{A} | \bar{E})$ its **specificity**, and $P(E | A)$ its **positive predictive value** (PPV). The current model has sensitivity **0.89** and specificity **0.90**.

Give every probability to at least four significant figures.

(a) The PPV of the current model: _____

(b) Upgrade A raises the sensitivity to **0.99** and leaves the specificity at **0.90**. Its PPV: _____

(c) Upgrade B leaves the sensitivity at **0.89** and raises the specificity to **0.994**. Its PPV: _____

(d) Keeping the sensitivity at **0.89**, what specificity would make the PPV exactly **0.5**? _____

Problem 13

seed 20260925

A payments processor scores each online card payment on two signals: S_1 , the payment comes from an unusual location for the cardholder, and S_2 , it comes from a device never seen before on the card. Let F be the event that the payment is fraudulent, with $P(F) = 0.003$, and

	$P(\cdot F)$	$P(\cdot \bar{F})$
S_1	0.83	0.035
S_2	0.77	0.055

Assume the two signals are conditionally independent given the state -- that is, given F , and again given $\bar{F}$, the events S_1 and S_2 are independent, so for example $P(S_1 \cap S_2 | F) = P(S_1 | F)P(S_2 | F)$. (They are **not** unconditionally independent: both are more common when F occurs.)

Give every probability to at least four significant figures.

(a) A payment shows S_1 . $P(F | S_1) =$ _____

(b) The same payment is then found to show S_2 as well. $P(F | S_1 \cap S_2) =$ _____

(c) If instead it had been found **not** to show S_2 , $P(F | S_1 \cap \bar{S}_2) =$ _____

Problem 14

seed 20260925

At month-end a bank's general ledger does not reconcile, and the break is known to sit in exactly one of three departments: treasury (B_1), payments (B_2) or trade finance (B_3). From past breaks,

$$P(B_1) = 0.39, \quad P(B_2) = 0.33, \quad P(B_3) = 0.28.$$

An internal audit of **treasury** finds the break with probability $1 - \alpha = 0.78$ if it is there (so it misses it with probability $\alpha = 0.22$), and of course cannot find it if it is not there. Let U be the event that the audit of treasury is unsuccessful.

Give every probability to at least four significant figures.

The audit of treasury is unsuccessful. What is the probability that the break is in

(a) treasury, $P(B_1 | U) =$ _____

(b) payments, $P(B_2 | U) =$ _____

(c) trade finance, $P(B_3 | U) =$ _____

(d) A second, separate audit of treasury is also unsuccessful. Assume that, given the break is in treasury, the two audits miss it independently, each with probability α . Now what is the probability that the break is in treasury?

Problem 15

seed 20260925

A credit bureau studies small-business borrowers. Among borrowers who went on to default, a fraction **0.45** had a record of at least one payment more than 30 days late in the previous two years. Among borrowers who did not default, the fraction with such a record was only **0.14**. The overall default rate is **0.022**.

Let D be the event that a randomly chosen borrower defaults and H the event that the borrower has a late-payment record. Give every probability to at least four significant figures.

(a) $P(H) =$ _____

(b) $P(D | H) =$ _____

(c) $P(D | \bar{H}) =$ _____

(d) The ratio $P(D | H)/P(D | \bar{H}) =$ _____

Problem 16

seed 20260925

Each quarter an equity analyst names which of the 10 sectors of the Baku stock-exchange index will perform best. In any quarter she has a genuinely informed view with probability **0.37**; when she does, her pick is right with probability **0.85**. Otherwise she guesses, choosing one of the 10 sectors at random, each equally likely.

Let I be the event that she has an informed view this quarter and R the event that her pick is right. Give every probability to at least four significant figures.

(a) $P(R) =$ _____

(b) Her pick turns out right. The probability that she was informed, $P(I | R) =$ _____

(c) Her pick turns out wrong. The probability that she was informed, $P(I | \bar{R}) =$ _____

Problem 17

seed 20260925

An analyst models next quarter's macro state as recession (B_1), stagnation (B_2) or expansion (B_3), with

$$P(B_1) = 0.13, \quad P(B_2) = 0.39, \quad P(B_3) = 0.48.$$

Let A be the event that a listed bank's share price falls over the quarter and C the event that a cement producer's share price falls. The analyst's conditional probabilities are

	B_1	B_2	B_3
$P(A B_i)$	0.70	0.53	0.19
$P(C B_i)$	0.77	0.46	0.22

and she assumes that **within each macro state** the two stocks move independently:

$P(A \cap C | B_i) = P(A | B_i)P(C | B_i)$ for $i = 1, 2, 3$. The macro state is a shared cause of both prices.

Give every probability to at least four significant figures.

(a) $P(A) =$ _____

(b) $P(A \cap C) =$ _____

(c) Both stocks fall. The probability that the quarter was a recession, $P(B_1 | A \cap C) =$ _____

(d) The bank stock falls. The probability that the cement stock also falls, $P(C | A) =$ _____

Problem 18

seed 20260925

A microfinance lender classifies each borrower as prime (B_1), near-prime (B_2) or subprime (B_3), but a new borrower's true class is unknown. Among new borrowers,

$$P(B_1) = 0.53, \quad P(B_2) = 0.29, \quad P(B_3) = 0.18.$$

Let M_1 and M_2 be the events that the borrower misses a scheduled payment in quarters 1 and 2. In each quarter the probability of a missed payment depends on the class only:

$$P(M_j | B_1) = 0.020, \quad P(M_j | B_2) = 0.11, \quad P(M_j | B_3) = 0.24, \quad j = 1, 2.$$

Assume M_1 and M_2 are conditionally independent given the class: for each i ,

$P(M_1 \cap M_2 | B_i) = P(M_1 | B_i)P(M_2 | B_i)$, and likewise for the complements.

Give every probability to at least four significant figures.

(a) The borrower misses in quarter 1. $P(B_3 | M_1) =$ _____

(b) The borrower misses in both quarters. $P(B_3 | M_1 \cap M_2) =$ _____

(c) Standing at the end of quarter 1, having seen only M_1 : the probability of a miss in quarter 2, $P(M_2 | M_1) =$ _____

(d) The borrower misses in quarter 1 but pays in quarter 2. $P(B_3 \mid M_1 \cap \bar{M}_2) =$ _____

seed 20260925

Problem 19

A retail bank studies which customers move their main account to a competitor within a year. Let M be the event that a randomly chosen customer uses the bank's mobile app, Y the event that the customer's salary is paid into the bank, and W the event that the customer switches banks. The two product events overlap:

$$P(M) = 0.45, \quad P(Y) = 0.58, \quad P(M \cap Y) = 0.20.$$

The bank's analysts report the switching rate for four customer groups:

group	$P(W \mid \text{group})$
app and salary	0.055
app, no salary	0.09
salary, no app	0.10
neither	0.23

Give every probability to at least four significant figures.

(a) The probability that a customer has neither product, $P(\bar{M} \cap \bar{Y}) =$ _____

(b) $P(W) =$ _____

(c) A customer switches. The probability that she had both products: _____

(d) A customer switches. The probability that he had neither product: _____

seed 20260925

Problem 20

A bank processes international payment orders at 4 regional back offices, each handling the same number of orders. Normally every office makes an error in a fraction **0.028** of orders, the errors scattered at random. In March a software fault at office 1 raised its error rate to **0.075**; the other offices were unaffected. The fault was discovered only later.

An auditor receives a batch of March orders, all from one office, but the batch label is lost, so each of the 4 offices is equally likely to be its source. She checks 5 orders from the batch. Let B be the event that the batch came from office 1 and A the event that **exactly one** of the 5 orders checked contains an error.

As in Example 2.23, the probability that exactly j of n orders contain an error, when each does so independently with probability p , is $\binom{n}{j} p^j (1-p)^{n-j}$. Give every probability to at least four significant figures.

(a) $P(A \mid B) =$ _____

(b) $P(A) =$ _____

(c) She finds exactly one error. $P(B \mid A) =$ _____

(d) Suppose instead she had found **exactly two** errors among the 5. Then the probability that the batch came from office 1 would be