

Week 5, Problem Set 1

Wackerly 2.11-2.13 - random variables and random sampling

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A bank's credit desk tracks three loans over the coming year: a consumer loan, an SME loan and a mortgage, issued to unrelated borrowers in different sectors. Treat their outcomes as independent. The one-year default probabilities are

$$P(\text{consumer defaults}) = 0.04, \quad P(\text{SME defaults}) = 0.14, \quad P(\text{mortgage defaults}) = 0.07.$$

A sample point records the outcome of each loan in the order (consumer, SME, mortgage), for example **DND** means the consumer loan and the mortgage default and the SME loan does not. Let **Y** be the number of loans that default.

Give every probability to at least four significant figures.

(a) $P(Y = 1) =$ _____

(b) $P(Y = 2) =$ _____

(c) $P(Y \geq 1) =$ _____

Problem 2

seed 20260925

An analyst models the direction of a stock index over the four trading days Monday to Thursday of one week. Each day the index closes either up (**U**) or down (**D**), and days are independent. The central bank makes an announcement before trading opens on Wednesday. On Monday and on Tuesday the probability of an up-day is **0.42**; from Wednesday onward (on Wednesday and on Thursday) it is **0.68**.

A sample point is a sequence of four letters such as **UDDU**. Let **Y** be the number of up-days in the week.

Give every probability to at least four significant figures.

(a) $P(Y = 2) =$ _____

(b) $P(Y = 3) =$ _____

(c) $P(Y \geq 3) =$ _____

Problem 3

seed 20260925

A trader holds two contracts that settle at the end of the month, one on Brent crude and one on gold. Treat their outcomes as independent.

- The oil contract pays off with probability **0.44**, giving a net gain of 340 AZN; otherwise the trader loses 250 AZN.
- The gold contract pays off with probability **0.66**, giving a net gain of 100 AZN; otherwise the trader loses 100 AZN.

Let Y be the trader's total net gain (in AZN) from the two contracts; a loss counts as a negative gain.

Give every probability to at least four significant figures.

(a) $P(Y = 240) =$ _____

(b) $P(Y > 0) =$ _____

(c) $P(Y > -250) =$ _____

Problem 4

seed 20260925

The Central Bank's supervision department audits 5 of a commercial bank's 10 branches this quarter. The branches to be audited are chosen by simple random sampling in the sense of Definition 2.13.

(a) How many different samples of branches are possible? _____

(b) How many of those samples include the Ganja branch? _____

(c) What is the probability that the Ganja branch is audited? Give at least four significant figures.

(d) Separately, the department inspects 4 of the 16 credit unions on its register, again by simple random sampling. How many different samples of credit unions are possible? _____

Enter whole numbers in (a), (b) and (d).

Problem 5

seed 20260925

A relationship manager has 12 corporate accounts and will phone two of them for a satisfaction survey. Two of the accounts belong to the firms Azmetal and Kurstrans.

Give every probability to at least four significant figures.

(a) The two accounts are chosen without replacement, all pairs being equally likely. What is the probability that the sample consists of Azmetal and Kurstrans? _____

(b) Instead, one account is drawn at random, put back, and a second is drawn at random from all 12. What is the probability that the two accounts drawn are Azmetal and Kurstrans (in either order)? _____

(c) Under the scheme in (b), what is the probability that the same account is drawn twice? _____

(d) A payments company picks three of its 38 merchants with replacement, each draw uniform over all 38. What is the probability that the three draws are three different merchants? _____

Problem 6

seed 20260925

Among the retail customers of a Baku bank, a fraction **0.35** hold a credit card, a fraction **0.35** hold a consumer loan, and a fraction **0.11** hold both. One customer is selected at random. Let Y be the number of these two products the customer holds, so Y takes the values 0, 1 and 2.

Give every probability to at least four significant figures.

(a) $P(Y = 0) =$ _____

(b) $P(Y = 1) =$ _____

(c) $P(Y = 2 \mid Y \geq 1) =$ _____

Problem 7

seed 20260925

A batch of 7 mortgage files is waiting for an internal audit. Exactly 3 of the files contain documentation errors, but nobody knows which. The auditor examines the files one at a time in a random order (all orderings equally likely) and stops as soon as the last flawed file has been found. Let Y be the number of the inspection at which that happens.

(a) What is the smallest possible value of Y ? _____

Give the probabilities to at least four significant figures.

(b) $P(Y = 4) =$ _____

(c) $P(Y \leq 5) =$ _____

(d) $P(Y = 7) =$ _____

(e) A second batch of 19 files contains exactly one flawed file, and the auditor has time for only 8 inspections, again in a random order. What is the probability that she finds the flawed file? _____

Problem 8

seed 20260925

A financial newspaper ranks 13 pension funds from 1 (best five-year return) to 13 (worst). A client who does not read the newspaper picks 5 of the funds at random, every set of 5 funds being equally likely. Let Y be the rank of the best fund in the client's selection (the smallest rank chosen).

(a) How many equally likely selections are there? Enter a whole number. _____

Give the probabilities to at least four significant figures.

(b) $P(Y = 2) =$ _____

(c) $P(Y \geq 7) =$ _____

Problem 9

seed 20260925

The State Tax Service selects 14 of the 31 firms in the construction sector for a detailed audit this year, by simple random sampling (Definition 2.13). Two of the firms are Qala Tikinti and Xazar Beton.

Give every probability to at least four significant figures.

(a) $P(\text{Qala Tikinti is audited}) =$ _____

(b) $P(\text{both Qala Tikinti and Xazar Beton are audited}) =$ _____

(c) $P(\text{at least one of the two firms is audited}) =$ _____

Problem 10

seed 20260925

A pension fund's bond portfolio holds 21 bonds, of which 10 are rated high yield (below investment grade). A risk officer reviews 4 bonds chosen at random. Let Y be the number of high-yield bonds among those reviewed.

Give every probability to at least four significant figures.

Sampling **without** replacement (a simple random sample of 4 different bonds):

(a) $P(Y = 0) =$ _____

(b) $P(Y = 2) =$ _____

Sampling **with** replacement (4 independent draws, each uniform over all 21 bonds, a bond possibly being drawn more than once; Y counts draws that give a high-yield bond):

(c) $P(Y = 0) =$ _____

(d) $P(Y = 2) =$ _____

Problem 11

seed 20260925

A currency desk places three trades on the EUR/USD rate during a day. Each trade wins with probability **0.50**, independently of the others (they are placed on different, unrelated signals and closed within minutes). A winning trade earns 50 AZN and a losing trade loses 60 AZN. Let Y be the desk's net gain from the three trades, in AZN.

Give every probability to at least four significant figures.

(a) $P(Y = -70) =$ _____

(b) $P(Y > 0) =$ _____

(c) $P(Y < 0) =$ _____

Problem 12

seed 20260925

A mobile operator sells three monthly tariffs: Basic at 9 AZN, Standard at 16 AZN and Premium at 23 AZN. A new subscriber chooses Basic with probability **0.30**, Standard with probability **0.38** and Premium with probability **0.32**. Two new subscribers, who do not know each other, sign up at a shop today; treat their choices as

independent. Let Y be the total monthly revenue (AZN) from these two subscribers.

Give every probability to at least four significant figures.

(a) $P(Y = 32) =$ _____

(b) $P(Y = 25) =$ _____

(c) $P(Y > 32) =$ _____

Problem 13

seed 20260925

A microfinance lender's scoring model flags loan applications as "high risk". Of all applicants, a proportion **0.04** would default if granted a loan. The model flags **0.93** of the applicants who would default and **0.11** of those who would not. An application is selected at random.

Give every probability to at least four significant figures.

(a) What is the probability that it is flagged? _____

(b) Given that it is flagged, what is the probability that the applicant would default? _____

(c) Given that it is not flagged, what is the probability that the applicant would default? _____

Problem 14

seed 20260925

A card-payment processor connects its Baku data centre to the interbank switch through four leased links. Links 1 and 2 are supplied by vendor X and each is up on a given day with probability **0.71**; links 3 and 4 are supplied by vendor Y, on a separate cable route, and each is up with probability **0.89**. The four links fail independently.

Two wiring designs are considered.

- **Design A:** two separate routes, one through links 1 then 2, the other through links 3 then 4. Traffic gets through if at least one route has both its links up.
- **Design B:** a first stage made of links 1 and 3 side by side, followed by a second stage made of links 2 and 4 side by side. Traffic gets through if at least one link in each stage is up.

Give every probability to at least four significant figures.

(a) $P(\text{traffic gets through under design A}) =$ _____

(b) $P(\text{traffic gets through under design B}) =$ _____

(c) Under design A, given that traffic gets through, what is the probability that links 1 and 4 are both up?

Problem 15

seed 20260925

A regional bond fund's approved list contains 5 Azerbaijani, 11 Turkish and 5 Kazakh sovereign and quasi-sovereign bonds, all different. For a new sub-portfolio the manager selects 4 bonds from the list by simple random sampling.

Give every probability to at least four significant figures.

(a) What is the probability that all three countries are represented? _____

(b) What is the probability that all four bonds come from the same country? _____

(c) Given that all three countries are represented, what is the probability that two of the four bonds are Azerbaijani? _____

Problem 16

seed 20260925

A motor insurer receives claims on policies sold through three channels. A proportion **0.43** of claims come from policies sold by agents, **0.33** from policies bought online and **0.24** from policies sold by brokers. The proportion of claims that turn out to be fraudulent is **0.020** for agent-sold policies, **0.075** for online policies and **0.035** for broker-sold policies. One claim is selected at random.

Give every probability to at least four significant figures.

(a) $P(\text{the claim is fraudulent}) =$ _____

(b) Given that the claim is fraudulent, what is the probability that the policy was bought online?

(c) Given that the policy was not sold by an agent, what is the probability that the claim is fraudulent?

Problem 17

seed 20260925

An equity analyst ranks the 10 listed bank stocks she covers from 1 (most attractive) to 10 (least). An investor who ignores the report buys 4 of the stocks chosen at random, every set of 4 being equally likely. Let R be the range of the ranks bought, $R = (\text{largest rank}) - (\text{smallest rank})$.

Give the probabilities to at least four significant figures.

(a) What is the probability that the best rank bought is 2 and the worst is 7? _____

(b) What is the smallest possible value of R ? _____

(c) $P(R = 5) =$ _____

(d) $P(R \leq 5) =$ _____

Problem 18

seed 20260925

A bank's Sumqayit branch holds 12 pending mortgage applications, 5 of them prime and 7 subprime. The Baku head office holds 5 prime and 10 subprime applications. One application is chosen at random at Sumqayit and sent to head office. A credit officer at head office then picks one application at random from everything now on her desk.

Give every probability to at least four significant figures.

(a) What is the probability that the officer picks a subprime application? _____

(b) Given that she picks a subprime application, what is the probability that the application sent from Sumqayit was prime? _____

(c) Given that she picks a prime application, what is the probability that the application sent from Sumqayit was prime? _____

Problem 19

seed 20260925

A proprietary trader's record over 11 trading days shows 7 gain days (**G**) and 4 loss days (**L**). If the trader's results carry no momentum, every arrangement of the 4 loss days among the 11 days is equally likely. A **run** is a maximal unbroken block of equal letters; for example **GGLLGG**... starts with a run of **G**s. Let **R** be the number of runs in the record.

Give every probability to at least four significant figures.

(a) What is the probability of the arrangement in which the 4 loss days are the last 4 days of the record?

(b) $P(R = 2) =$ _____

(c) $P(R \leq 3) =$ _____

(d) A second trader's record covers 13 days with 5 loss days, again with all arrangements equally likely. What is the probability that the 5 loss days are consecutive (form a single block)? _____

Problem 20

seed 20260925

A commodities desk models Brent crude over the next two trading days. On day 1 the price rises with probability **0.37** and otherwise falls; on day 2 it rises with probability **0.66** and otherwise falls. The two days' moves are independent. Define the events

A: the price rises on day 1.

B: the price rises on day 2.

C: the price moves in the same direction on both days.

Give the probabilities to at least four significant figures.

(a) $P(C) =$ _____

(b) $P(A \cap C) =$ _____

(c) Are **A** and **C** independent?

☐ Independent

☐ Dependent

(d) Are B and C independent?

- ☐ Independent
- ☐ Dependent

(e) Are A , B and C mutually independent (all three pairwise product rules and $P(A \cap B \cap C) = P(A)P(B)P(C)$)?

- ☐ Yes
- ☐ No