Week 5, Problem Set 2

Wackerly 3.1-3.3 - discrete random variables and expected value

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A Baku microfinance lender has four small loans outstanding to market traders. Let Y be the number of these loans that default within the year. The lender's risk team models Y with the probability distribution below, but the entry for $y = 0$ has been left blank as an unknown constant c.

y	0	1	2	3	4
$p(y)$	c	0.20	0.14	0.03	0.03

Give every probability to at least four significant figures.

(a) The value of c that makes this a valid probability distribution: $c =$ _____

(b) The probability that at least two loans default: $P(Y \geq 2) =$ _____

(c) $P(1 \leq Y \leq 3) =$ _____

Problem 2

seed 20260925

A mobile operator's call centre in Baku logs complaints about dropped calls. Let Y be the number of such complaints received in a randomly chosen fifteen-minute interval. Its probability distribution is

y	0	1	2	3	4	5
$p(y)$	0.05	0.18	0.41	0.17	0.14	0.05

and $p(y) = 0$ for every other y.

Give every probability to at least four significant figures.

(a) $P(1 \leq Y \leq 4) =$ _____

(b) $P(Y > 4) =$ _____

(c) $P(1 < Y < 4) =$ _____

Problem 3

seed 20260925

An exporter in Ganja sends a payment abroad through its bank. Let Y be the number of business days until the payment settles. The bank's operations team models Y with the probability function

$$p(y) = c\,(14 - y), \qquad y = 1, 2, \ldots, 4,$$

and $p(y) = 0$ otherwise, where c is a constant.

Give every answer to at least four significant figures (a fraction such as $1/37$ is also accepted in (a)).

(a) $c =$ _____

(b) $P(Y \le 2) =$ _____

(c) $E(Y) =$ _____ business days

(d) While the payment is in transit the exporter loses 9 AZN per business day in interest. The expected interest cost of one payment is _____ AZN.

Problem 4

seed 20260925

An insurer in Baku sells compulsory motor third-party liability cover. For a randomly chosen policy let Y be the number of claims filed during the policy year. From past portfolio data,

y	0	1	2	3
$p(y)$	0.83	0.10	0.06	0.01

Each claim costs the insurer 2200 AZN on average, and for this problem treat every claim as costing exactly that amount.

(a) The expected number of claims per policy: $E(Y) =$ _____

(b) The expected claim cost per policy: _____ AZN

(c) The probability that a policy produces at least one claim: _____

Problem 5

seed 20260925

An agricultural insurer offers vineyard owners near Shamakhi a one-year hail policy that pays 5500 AZN if hail destroys the harvest and nothing otherwise. Historically a hail loss of this kind occurs with probability 0.047 per vineyard per year. Handling each policy costs the insurer 20 AZN in administration, whether or not a claim is paid. The insurer's profit on a policy is what remains of the premium after payouts and administration.

(a) The expected payout on one policy: _____ AZN

(b) The premium C the insurer must charge so that its expected profit per policy is 75 AZN: $C =$ _____ AZN

(c) The sales team proposes a premium of 285 AZN instead. The expected profit per policy at that premium is _____ AZN (enter a negative number for an expected loss).

Problem 6

seed 20260925

A household in Sheki receives a monthly remittance of Y US dollars from a relative working abroad, where $E(Y) = 420$ and the standard deviation of Y is 285 USD. The receiving bank converts the transfer at a fixed rate of 1.69 AZN per USD and deducts a flat fee of 9 AZN, so the household is credited

$$X = 1.69\,Y - 9 \quad \text{AZN}.$$

Give every answer to at least four significant figures.

(a) $E(X) =$ _____ AZN

(b) $V(X) =$ _____ AZN^2

(c) The standard deviation of X: _____ AZN

Problem 7

seed 20260925

A pension fund holds four corporate bonds. Let Y be the number of them whose credit rating is downgraded during the next year. The fund's credit analysts use the distribution

y	0	1	2	3	4
$p(y)$	0.52	0.22	0.18	0.05	0.03

Give every answer to at least four significant figures.

(a) $\mu = E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $\sigma^2 = V(Y) =$ _____

(d) $\sigma =$ _____

Problem 8

seed 20260925

A venture-debt fund lends to a Baku software start-up. The loan is repaid in a single payment of 190 thousand AZN at the end of year Y, where the fund models the repayment year as

y	1	2	3	4
$p(y)$	0.18	0.35	0.19	0.28

The fund discounts at 10% per year, so the present value of the repayment is

$$g(Y) = \frac{190}{(1.10)^{Y}} \quad \text{thousand AZN}.$$

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ years

(b) The expected present value $E[g(Y)] =$ _____ thousand AZN

(c) The present value computed at the mean repayment year, $g(E(Y)) = 190/(1.10)^{E(Y)} =$ _____ thousand AZN

Problem 9

seed 20260925

A bank's credit committee has 11 loan applications on the table this week: 2 from small and medium-sized enterprises (SMEs) and 9 from large corporates. It has time to review only 3, and to avoid any appearance of favouritism it draws the three at random, every set of three being equally likely. Let Y be the number of SME applications among the three drawn.

Give every answer to at least four significant figures (fractions are accepted).

(a) $p(0) = P(Y = 0) =$ _____

(b) $p(1) =$ _____

(c) $E(Y) =$ _____

(d) A state programme pays the bank a subsidy of 450 AZN for every SME application that is reviewed. The expected subsidy from this week's draw is _____ AZN.

Problem 10

seed 20260925

A carpet exporter in Guba sends cross-border wire transfers to suppliers. Let Y be the number of transfers it sends in a randomly chosen week:

y	1	2	3	4	5
$p(y)$	0.12	0.30	0.34	0.14	0.10

Its bank charges a fixed account fee of 7 AZN per week plus 5.50 AZN per transfer, so the weekly bank charge is $C = 7 + 5.50\,Y$ AZN.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) $E(C) =$ _____ AZN

(d) The standard deviation of C: _____ AZN

Problem 11

seed 20260925

An investment committee in Baku must fund one of two expansion projects. Their one-year returns (in percent) depend on the state of the regional economy:

State	Recession	Normal	Boom
Probability	0.18	0.53	0.29
Return of A, R_A (%)	-4	8	13
Return of B, R_B (%)	1	7	16

Give every answer to at least four significant figures.

(a) $E(R_A) =$ _____ % $E(R_B) =$ _____ %

(b) Standard deviations: $\sigma_A =$ _____ % $\sigma_B =$ _____ %

(c) An investor prefers a higher expected return and a lower standard deviation. Does one project dominate the other on both counts?

- ○ Project A: higher expected return and lower standard deviation
- ○ Project B: higher expected return and lower standard deviation
- ○ Neither: the project with the higher expected return is also the riskier one

Problem 12

seed 20260925

A freight forwarder tracks how long import containers wait for customs clearance at the Port of Baku in Alat. Let Y be the waiting time, in whole days, of a randomly chosen container:

y	1	2	3	4	5	6	7
$p(y)$	0.06	0.25	0.23	0.23	0.14	0.05	0.04

Give every answer to at least four significant figures.

(a) $\mu = E(Y) =$ _____ days

(b) $\sigma =$ _____ days

(c) The probability that Y falls strictly inside one standard deviation of the mean, $P(\mu - \sigma < Y < \mu + \sigma) =$ _____

(d) $P(\mu - 2\sigma < Y < \mu + 2\sigma) =$ _____

Problem 13

seed 20260925

An insurer in Baku writes one-year fire policies on warehouses, each insured for 85000 AZN. From its loss history, a warehouse of this type is totally destroyed in a year with probability 0.0040 and suffers a partial loss of 60% of its value with probability 0.011. Ignore all other partial losses. Let X be the insurer's payout on one policy.

Give every answer to at least four significant figures.

(a) The premium at which the insurer breaks even on average, $E(X) =$ _____ AZN

(b) Each policy also costs 40 AZN to administer, and the insurer wants an expected profit of 140 AZN per policy. The premium it should charge is _____ AZN

(c) The standard deviation of the payout, $\sigma_X =$ _____ AZN

Problem 14

seed 20260925

A data centre in Sumgait that hosts servers for several banks records Y, the number of unplanned power outages in a month:

y	0	1	2	3
$p(y)$	0.52	0.23	0.20	0.05

Because repeated outages trigger penalty clauses in its client contracts, the monthly cost of running the backup system is

$$C = 1400 + 400\,Y + 275\,Y^2 \quad \text{AZN}.$$

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $E(C) =$ _____ AZN

Problem 15

seed 20260925

An analyst models next year's consumer-price inflation in Azerbaijan, Y (in percent), as a discrete random variable over a grid of scenarios, with $E(Y) = 4.4$ and standard deviation 2.8. The central bank's target is 3.5%, and a common way to score a policy outcome is the squared miss $(Y - 3.5)^2$.

Give every answer to at least four significant figures.

(a) $E(Y^2) =$ _____

(b) The expected squared miss from the target, $E[(Y - 3.5)^2] =$ _____

(c) Of all constants c, the one that makes $E[(Y - c)^2]$ as small as possible is $c =$ _____, and the smallest possible value of $E[(Y - c)^2]$ is _____

Problem 16

seed 20260925

A bond fund holds one bond from each of three issuers: a Georgian utility, a Kazakh telecom operator and a Turkish retailer. Over the next year the probabilities that each issuer defaults are 0.03, 0.12 and 0.08 respectively, and the fund's analysts treat the three defaults as independent events. Let Y be the number of the three bonds that default.

Give every probability to at least four significant figures.

(a) $p(0) =$ _____

(b) $p(1) =$ _____

(c) $P(Y \geq 2) =$ _____

(d) $E(Y) =$ _____

Problem 17

seed 20260925

The cash-processing centre of a Baku bank is choosing between two banknote sorting machines. If a machine runs for t hours in a day, the number of jams Y that need an engineer is a random variable with **mean and variance both equal to λt**, where $\lambda_A = 0.12$ for machine A and $\lambda_B = 0.07$ for machine B. Because each extra call-out in a day is charged at a rising rate, the daily costs (AZN) are

$$C_A(t) = 4.06\,t + 40\,Y_A^2, \qquad C_B(t) = 11.00\,t + 40\,Y_B^2.$$

Assume each machine starts every day as good as new.

Give every number to at least four significant figures.

(a) For a working day of 10 hours: $E[C_A(10)] =$ _____ AZN and $E[C_B(10)] =$ _____ AZN

(b) Which machine has the lower expected daily cost on a day of 10 hours?

- ○ Machine A
- ○ Machine B

(c) Which machine has the lower expected daily cost on a day of 23 hours?

- ○ Machine A
- ○ Machine B

(d) The length of day $t > 0$ at which the two expected costs are equal: $t =$ _____ hours

Problem 18

seed 20260925

A corporate-banking relationship manager in Baku meets either one client a day, with probability 0.25, or two clients, with probability 0.75. Each meeting, independently of the others, ends either with a new credit line of 50 thousand AZN, with probability 0.28, or with nothing, with probability 0.72. Let X be the total value of new credit lines arranged in a day (thousand AZN), so X takes the values 0, 50 and 100.

Give every answer to at least four significant figures.

(a) $P(X = 0) =$ _____

(b) $P(X = 100) =$ _____

(c) $E(X) =$ _____ thousand AZN

(d) The standard deviation of X: _____ thousand AZN

Problem 19

seed 20260925

A bank tracks how the internal credit ratings of its corporate borrowers move over a year. For a randomly chosen borrower let Y be the change in rating, in notches (positive for an upgrade, negative for a downgrade). The bank's model is

y	-2	-1	0	1	2
$p(y)$	0.04	a	0.59	b	0.04

where a and b were lost when the report was reformatted. The report still states that the mean rating change is $E(Y) = -0.05$ notches.

Give every answer to at least four significant figures.

(a) $a = P(Y = -1) =$ _____

(b) $b = P(Y = 1) =$ _____

(c) $V(Y) =$ _____

Problem 20

seed 20260925

An insurer has written a large property policy on a shopping centre in Baku. Its loss Y on this policy over the year (thousand AZN) is modelled as

y	0	25	150	275
$p(y)$	0.77	0.15	0.05	0.03

To cap its exposure it buys an **excess-of-loss** reinsurance contract with a retention of 75 thousand AZN: the reinsurer pays whatever part of the loss exceeds 75, that is

$$g(Y) = \max(Y - 75,\ 0).$$

Give every answer to at least four significant figures.

(a) The insurer's expected loss before reinsurance, $E(Y) =$ _____ thousand AZN

(b) The reinsurer's expected payout, $E[g(Y)] =$ _____ thousand AZN

(c) The standard deviation of the reinsurer's payout: _____ thousand AZN

(d) The reinsurer charges its expected payout plus a safety loading of 0.35 times the expected payout. The premium is _____ thousand AZN