

Week 6, Problem Set 1

Wackerly 3.4 - The Binomial Probability Distribution

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A bank's card department offers a free upgrade to a premium card to 8 existing customers, chosen at random from its customer base. Each customer accepts with probability **0.38**, independently of the others. Let **Y** be the number of the 8 customers who accept.

Give every probability to at least four significant figures.

(a) $P(Y = 3) =$ _____

(b) What is the probability that nobody accepts? _____

Problem 2

seed 20260925

A courier company in Baku insures its fleet of 14 delivery vans with one insurer. The vans work separate routes, and the insurer's actuary models each van as filing at least one claim during the policy year with probability **0.32**, independently of the other vans. Let **Y** be the number of vans that file a claim.

Give each answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) The standard deviation of **Y** is _____

(d) The expected number of vans that file **no** claim is _____

Problem 3

seed 20260925

An external auditor is checking the invoice file of a large wholesale distributor, which contains tens of thousands of invoices. Past work suggests that 15.0% of the invoices carry a pricing error. The auditor selects 10 invoices at random. Let **Y** be the number of sampled invoices with a pricing error; the file is so large that **Y** may be treated as binomial.

Give every probability to at least four significant figures.

(a) What is the probability that the sample contains at least one invoice with an error? _____

(b) What is the probability that it contains at least two? _____

Problem 4

seed 20260925

For each situation decide whether Y is a binomial random variable in the sense of Definition 3.6. Where the sampling is from a very large population, treat “approximately binomial” (as in Example 3.6) as binomial.

(a) A mobile operator texts a retention offer to 25 subscribers chosen at random from its 3 million subscribers. Y = the number of the 25 who accept the offer.

- ☐ Binomial
- ☐ Not binomial: the number of trials is not fixed in advance
- ☐ Not binomial: the trials are not independent, or p is not the same on every trial
- ☐ Not binomial: Y is not a count of successes in two-outcome trials

(b) Twelve exporters each report their percentage revenue growth for the year. Y = the average of the twelve growth rates.

- ☐ Binomial
- ☐ Not binomial: the number of trials is not fixed in advance
- ☐ Not binomial: the trials are not independent, or p is not the same on every trial
- ☐ Not binomial: Y is not a count of successes in two-outcome trials

(c) A food exporter in Guba ships 13 consignments by unrelated carriers on unrelated routes; each arrives late with probability **0.23**, independently. Find the probability that at most one consignment is late, to at least four significant figures. _____

Problem 5

seed 20260925

A microfinance lender receives 12 small-business loan applications this week from unrelated applicants. Each application arrives with the full set of documents with probability **0.87**, independently of the others. Let Y be the number of complete applications.

Give every probability to at least four significant figures (a calculator or R's `pbinom` is fine).

(a) $P(Y \leq 9) =$ _____

(b) $P(Y > 9) =$ _____

(c) $P(Y < 9) =$ _____

Problem 6

seed 20260925

A pension fund holds bonds issued by 10 unrelated companies in different industries. Over the next year each issuer defaults with probability **0.140**, independently of the others. The fund's report tracks Y = the number of the 10 bonds that are **repaid in full** (do not default).

Give each answer to at least four significant figures.

(a) Y is binomial. What is its success probability p ? _____

(b) $P(Y = 8) =$ _____

(c) $E(Y) =$ _____

(d) $V(Y) =$ _____

Problem 7

seed 20260925

A bank's mobile app lets new customers open an account remotely. Of the 16 people who started onboarding this morning, each finishes the process within 24 hours with probability **0.82**, independently of the others. Let Y be the number who finish within 24 hours.

Give every probability to at least four significant figures.

(a) Exactly 13 finish. _____

(b) At least 10 finish. _____

(c) At least 9 but not more than 13 finish. _____

(d) Fewer than 9 finish. _____

Problem 8

seed 20260925

An energy company has raised enough capital to drill 7 exploration wells in separate prospects on the Caspian shelf. Each well is commercially successful with probability **0.28**, independently of the others. Before the first well the company pays a fixed 22 million AZN to mobilise the drilling rig. Each successful well then costs 43 million AZN (drilling plus completion) and each dry well 17 million AZN. Let Y be the number of successful wells and C the total cost, in million AZN, of the programme.

Give each answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(C) =$ _____ million AZN

(c) $V(C) =$ _____ (million AZN)²

(d) The standard deviation of C is _____ million AZN

Problem 9

seed 20260925

A distributor sells 5 card-payment terminals to a supermarket chain, taken at random from a large warehouse stock in which a proportion **0.19** of the units are defective. The chain returns every defective unit for repair. If Y is the number of defective terminals among the 5, the distributor's repair cost, in tens of AZN, is

$$C = 3Y^2 + 5Y + 6.$$

(The quadratic term reflects overtime once several units arrive together.)

Give each answer to at least four significant figures.

(a) $E(Y^2) =$ _____

(b) The expected repair cost $E(C) =$ _____ (tens of AZN)

(c) A junior analyst “estimates” the expected cost by plugging $E(Y)$ into the cost formula. By how much does $E(C)$ exceed that plug-in value? _____

Problem 10

seed 20260925

An importer sells a shipment of 10 Wi-Fi routers to an internet provider at 130 AZN each. To win the contract it offers a double-your-money-back guarantee: for every defective router the provider receives 2×130 AZN back. Each router is defective with probability **0.12**, independently of the others. Let G be the importer’s net gain, in AZN, from the shipment (sales revenue minus refunds).

Give each answer to at least four significant figures.

(a) $E(G) =$ _____ AZN

(b) The standard deviation of G is _____ AZN

(c) What is the probability that the net gain is less than 910 AZN? _____

Problem 11

seed 20260925

A bank’s compliance officer audits a portfolio of 35 business accounts, of which 14 are flagged for incomplete customer-identification (KYC) documents. She selects 10 accounts at random **without replacement** and records Y = the number of flagged accounts in her sample.

Give every probability to at least four significant figures.

(a) The probability that the first account drawn is flagged. _____

(b) The probability that the second account drawn is flagged, given that the first one was flagged.

(c) Is the binomial distribution with $n = 10$ and p equal to your answer to (a) an appropriate model for Y ?

☐ Yes: the sample is a small fraction of the portfolio, so the draws are nearly independent and Y is approximately binomial

☐ No: the sample is a large fraction of the portfolio, so the chance of a flagged account changes noticeably from draw to draw

☐ Yes: sampling without replacement always produces a binomial count

☐ No: each sampled account has more than two possible outcomes

(d) Using that binomial model regardless, compute $P(Y = 0)$. _____

(e) Compute the exact $P(Y = 0)$ for sampling without replacement, using the multiplicative law draw by draw.

Problem 12

seed 20260925

A leasing company has 18 equipment leases with unrelated small firms. Each lease is paid to term with probability **0.95**, independently of the others. Let Y be the number of leases that default and $Y^* = 18 - Y$ the number paid to term.

Printed binomial tables often stop at $p = 0.5$ or list only a few values of p , so the risk analyst wants every answer expressed through Y , whose success probability is small.

Give each answer to at least four significant figures.

(a) $P(Y^* \geq 16)$ _____

(b) $P(Y^* = 18)$ _____

(c) $V(Y^*)$ _____

(d) $E(Y)$ _____

Problem 13

seed 20260925

A fintech company signs up shops for its QR-code payment service. A sales agent visits shops one at a time; each shop visited signs up with probability **0.12**, independently of the others. The agent is paid a commission of 35 AZN per shop signed. The regional manager wants to plan a day of visits so that the probability of **at least one** sign-up is at least **0.98**.

(a) What is the smallest number of shops the agent must visit? _____

(b) With that number of visits, what is the probability of at least one sign-up? (at least four significant figures)

(c) With that number of visits, what is the agent's expected commission for the day, in AZN?

Problem 14

seed 20260925

A pension fund is reviewing an equity manager who beat her benchmark (after fees) in 8 of the last 14 months. The fund's analysts calculate that a manager with **no skill** would beat this benchmark in any given month with probability **0.47**, independently from month to month. Let Y be the number of months out of 14 in which a no-skill manager beats the benchmark.

(a) The expected number of months in which a no-skill manager beats the benchmark. _____

(b) The probability that a no-skill manager beats the benchmark in **at least** 8 of the 14 months (at least four significant figures). _____

(c) Using 0.05 as the dividing line, is her record unusual for a manager with no skill?

☐ Yes: under the no-skill model a record this good has probability below 0.05, so the no-skill explanation is hard to believe

☐ No: under the no-skill model a record at least this good has probability 0.05 or more, so it is not unusual for a

manager with no skill

Problem 15

seed 20260925

A bank is considering a mobile savings “pot” product. It surveys 18 customers chosen at random from its large retail base, and 12 say they would use it. Let p be the true proportion of the bank’s customers who would use the product, so that the number in the sample who say yes is binomial with $n = 18$ and unknown p .

Give each answer to at least four significant figures.

(a) The value of p that maximises the probability of observing exactly 12 yes answers. _____

(b) That maximised probability, $P(Y = 12)$ evaluated at your answer to (a). _____

(c) The product manager claims $p = 0.35$. Evaluate $P(Y = 12)$ at $p = 0.35$. _____

(d) How many times more probable is the observed result under the estimate from (a) than under the manager’s claim? (the ratio of (b) to (c)) _____

Problem 16

seed 20260925

A boutique hotel in Sheki has 14 rooms. For a festival weekend it accepts 18 non-refundable one-room bookings, 4 more than it has rooms, because some guests never arrive. Each booked guest arrives with probability **0.84**, independently of the others (the bookings come from unrelated travellers). Let Y be the number of guests who arrive.

Give each answer to at least four significant figures.

(a) The probability that more guests arrive than there are rooms. _____

(b) $E(Y) =$ _____

(c) The expected number of arriving guests who cannot be given a room. _____

(d) The expected number of rooms left empty. _____

Problem 17

seed 20260925

A bank is considering buying a pool of several thousand consumer loans from a microfinance originator. Before signing, its credit-risk team audits a random sample of loan files for documentation defects and buys the pool only if the number of defective files in the sample is small. Two audit plans are on the table:

- **Plan 1:** audit 11 files; buy if at most 1 is defective.
- **Plan 2:** audit 24 files; buy if at most 3 are defective.

The bank calls the pool **good** if **0.03** of its files are defective and **bad** if **0.19** are. The pool is so large that the number of defective files in the sample may be treated as binomial.

Give every probability to at least four significant figures.

(a) The probability that the bank buys a **good** pool under Plan 1 _____ and under Plan 2 _____.

(b) The probability that the bank buys a **bad** pool under Plan 1 _____ and under Plan 2 _____.

(c) Which plan gives the bank the smaller probability of buying a bad pool?

- ☐ Plan 1
☐ Plan 2

(d) Which plan gives the smaller probability of wrongly rejecting a good pool (the risk borne by an honest originator)?

- ☐ Plan 1
☐ Plan 2

Problem 18

seed 20260925

A regional airline flies a 19-seat turboprop between Baku and Lankaran. A ticket costs 115 AZN and is non-refundable, so the airline keeps the fare whether or not the passenger shows up. Each ticket holder shows up with probability **0.91**, independently of the others. If more ticket holders show up than there are seats, each passenger who cannot board is paid 250 AZN compensation. The airline will sell between 19 and 24 tickets and wants to maximise its expected revenue net of compensation.

Let Y_n be the number of ticket holders who show up when n tickets are sold. Give each answer to at least four significant figures.

(a) If 21 tickets are sold, the expected number of passengers who cannot board. _____

(b) If 21 tickets are sold, the expected net revenue, in AZN. _____

(c) The number of tickets, between 19 and 24, that maximises expected net revenue. _____

(d) That maximum expected net revenue, in AZN. _____

Problem 19

seed 20260925

A bank packages 22 loans of 1 million AZN each, made to unrelated firms in different sectors and regions, into a pool and sells it to investors in three slices (tranches). Over the coming year each loan defaults with probability **0.095**, independently of the others, and each default costs the pool **0.60** million AZN. Losses are absorbed in order of seniority:

- the **junior** tranche absorbs the losses from the first 2 defaults;
- the **middle** tranche absorbs the losses from the next 4 defaults (defaults number 3 to 6);
- the **senior** tranche absorbs any losses beyond 6 defaults.

Let Y be the number of defaults. Give every answer to at least four significant figures.

(a) The probability that the pool suffers no default.

(b) The probability that the junior tranche is wiped out completely.

(c) The probability that the senior tranche loses anything.

(d) The expected loss of the middle tranche, in million AZN.

Problem 20

seed 20260925

A bank's Ganja branch must fill 3 teller posts before a new service hall opens. It makes job offers to shortlisted candidates, each of whom accepts with probability **0.68**, independently of the others, and it will take on every candidate who accepts. HR wants the number of offers to be the smallest that makes the probability of at least 3 acceptances at least **0.90**. Each accepted offer costs 375 AZN in training and onboarding.

Give each probability to at least four significant figures.

(a) The number of offers HR should make.

(b) With that many offers, the probability of at least 3 acceptances.

(c) With one offer fewer, the probability of at least 3 acceptances.

(d) With the number of offers in (a), the expected onboarding cost, in AZN.

