

Week 6, Problem Set 2

Wackerly 3.5-3.6 - The Geometric and Negative Binomial Distributions

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A telesales agent for a Baku insurance company cold-calls small businesses to sell a property-insurance policy. Past campaigns show that a single call ends in a sale with probability $p = 0.09$, and calls are to randomly chosen firms, so outcomes are independent from call to call. Let Y be the number of the call on which the agent makes her **first** sale of the day.

Give every probability to at least four significant figures.

(a) $P(Y = 8) =$ _____

(b) The probability that the first sale comes within the first 8 calls, $P(Y \leq 8) =$ _____

(c) The expected number of calls needed to make the first sale, $E(Y) =$ _____

Problem 2

seed 20260925

The internal-audit unit of a large bank checks employee expense claims drawn at random from a very large pool. A fraction $p = 0.060$ of claims are fraudulent, and claims are drawn independently. Auditors keep checking claims until they find the first fraudulent one; let Y be the number of the audit on which that happens.

Give every probability to at least four significant figures.

(a) The probability that the first 15 audits find no fraud at all, $P(Y > 15) =$ _____

(b) The probability that the first fraudulent claim is found on or after audit number 5, $P(Y \geq 5) =$ _____

(c) The probability that the first fraudulent claim is found somewhere from audit 16 to audit 20 inclusive, $P(15 < Y \leq 20) =$ _____

Problem 3

seed 20260925

A bank's cash-logistics team models one busy ATM in Nasimi district. On any given day, independently of other days, the machine runs out of cash before the evening refill with probability $p = 0.019$. Let Y be the number of the day on which the first stock-out occurs (day 1 is tomorrow).

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ days

(b) The standard deviation of Y , $\sigma =$ _____ days

(c) $P(\mu - \sigma < Y < \mu + \sigma) =$ _____

(d) The probability of at least one stock-out in the next 28 days, $P(Y \leq 28) =$ _____

seed 20260925

Problem 4

A car-leasing company's credit committee reviews applications one at a time. Each application is approved with probability $p = 0.62$, independently of the others. The branch needs 2 approved leases to fill this week's delivery of vehicles, and reviews applications until it has them. Let Y be the number of the application on which the 2nd approval occurs.

Give every answer to at least four significant figures.

(a) $P(Y = 4) =$ _____

(b) $E(Y) =$ _____

(c) Let $Y^* = Y - 2$ be the number of **rejected** applications reviewed before the 2nd approval. $E(Y^*) =$ _____

Problem 5

seed 20260925

A mobile operator's retention team phones subscribers who have asked to port their number to a competitor. Each call, independently, ends with the subscriber accepting a discounted tariff with probability $p = 0.38$. This month the team has a target of 4 retained subscribers and keeps calling until it reaches it. Each call costs the operator 3.50 AZN in agent time. Let Y be the number of calls made and $C = 3.50 Y$ the total calling cost.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) $E(C) =$ _____ AZN

(d) The standard deviation of C is _____ AZN

Problem 6

seed 20260925

An exporter of Ganja wine files customs declarations one shipment at a time. The customs risk engine sends each declaration to physical inspection with probability $p = 0.12$, independently of past selections. Let Y be the number of the declaration that is the first one sent to inspection.

Give every answer to at least four significant figures.

(a) $P(Y > 10) =$ _____

(b) The first 10 declarations have all cleared without inspection. Find the probability that the next 3 also clear, $P(Y > 13 \mid Y > 10) =$ _____

(c) Given that the first 10 declarations cleared, the expected number of **additional** declarations up to and including the first inspection, $E(Y - 10 \mid Y > 10) =$ _____

Problem 7

seed 20260925

A credit analyst models a BB-rated Caspian shipping company as defaulting in any given year with probability $p = 0.024$, provided it has not defaulted before, independently from year to year. Let Y be the number of the year in which the company first defaults (year 1 is the coming year).

(a) The company has just issued a 10-year bond. Find the probability that it survives to maturity without defaulting, $P(Y > 10) =$ _____ (at least four significant figures)

(b) Find the largest integer y_0 such that $P(Y > y_0) \geq 0.65$. $y_0 =$ _____

(c) Find the smallest integer n such that the probability of a default within the first n years is at least 0.45, that is $P(Y \leq n) \geq 0.45$. $n =$ _____

Problem 8

seed 20260925

A manufacturing firm in Sumgait is moving its payroll account. Two banks, A and B, take turns presenting offers: bank A pitches in weeks 1, 3, 5, ... and bank B in weeks 2, 4, 6, Each pitch, whoever makes it, persuades the firm with probability $p = 0.170$, independently of earlier pitches, and the firm signs with the first bank whose pitch succeeds. Let Y be the week in which the account is won.

Give every probability to at least four significant figures.

(a) The probability that bank A wins the account, _____

(b) Given that bank B wins the account, the probability that B won it with its **second** pitch (week 4),

(c) The probability that the account is still unsigned after 14 weeks, $P(Y > 14) =$ _____

Problem 9

seed 20260925

An early-stage venture fund in Baku screens startup applications one at a time. Each screening costs the fund 800 AZN in analyst time, and a screened startup passes to the investment committee with probability $p = 0.08$, independently of the others. The fund stops at the first startup that passes and then commissions a legal and financial deep-dive on it costing 3000 AZN. Let Y be the number of startups screened and $T = 800Y + 3000$ the total cost of finding one investable startup.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(T) =$ _____ AZN

(c) The standard deviation of T is _____ AZN

(d) This quarter the fund can afford at most 14 screenings. The probability that none of them passes,

Problem 10

seed 20260925

A microfinance lender's collections agent works through a randomly ordered list of overdue borrowers, phoning each once, and stops as soon as a borrower agrees to a restructured repayment plan. Model the number of the call on which the first agreement happens as geometric with an unknown agreement probability p . Today the first agreement came on call number $y_0 = 5$.

Give every answer to at least four significant figures.

(a) The maximum-likelihood estimate of p , $\hat{p} =$ _____

(b) The value of the likelihood at the estimate, $L(\hat{p}) = P(Y = 5 \mid p = \hat{p}) =$ _____

(c) The branch manager insists the agreement rate is really $p = 0.47$. Find the likelihood ratio $L(0.47)/L(\hat{p})$.

(d) Using $\hat{p}$ as if it were the true value, estimate the probability that tomorrow the agent needs more than 4 calls to get the first agreement. _____

Problem 11

seed 20260925

A Baku bank is arranging a syndicated loan for a cement producer and needs 2 other banks to take a participation. It approaches banks one at a time from a long list; each bank approached agrees to join with probability $p = 0.575$, independently of the others. Let Y be the number of the approach on which the 2nd participant signs up.

Give every answer to at least four significant figures.

(a) $P(Y = 5) =$ _____

(b) The probability that the syndicate is complete within 5 approaches, $P(Y \leq 5) =$ _____

(c) $E(Y) =$ _____

Problem 12

seed 20260925

The SME desk of a Baku bank approves each small-business loan application with probability $p = 0.26$, independently of all other applications, and reviews applications in the order they arrive. Two questions are asked about the stream of decisions starting tomorrow morning.

Give every probability to at least four significant figures.

(a) The event is: the third approval is application number 14.

Which distribution describes the random variable that this event is about?

- ☐ Binomial (Definition 3.7)
- ☐ Geometric (Definition 3.8)
- ☐ Negative binomial (Definition 3.9)

Its probability is _____

(b) The event is: exactly 3 of the next 14 applications are approved.

Which distribution describes the random variable that this event is about?

- ☐ Binomial (Definition 3.7)
- ☐ Geometric (Definition 3.8)
- ☐ Negative binomial (Definition 3.9)

Its probability is _____

Problem 13

seed 20260925

A fixed-income trading desk has a daily loss limit. The risk department models each trading day as breaching the limit with probability $p = 0.035$, independently of every other day. Let Y be the number of the trading day on which the first breach occurs, counting from the start of the quarter. The desk has now completed 19 trading days without a breach.

Give every answer to at least four significant figures.

(a) The probability that the desk also gets through the next 6 days without a breach, $P(Y > 25 \mid Y > 19) =$ _____

(b) The probability that the first breach is on day 24, $P(Y = 24 \mid Y > 19) =$ _____

(c) $E(Y \mid Y > 19) =$ _____

(d) For comparison, the probability computed at the **start** of the quarter that the first 25 days would all be clean, $P(Y > 25) =$ _____

Problem 14

seed 20260925

A payments company is rolling out card terminals to shops in a new Baku shopping district, and a sales representative must sign up 3 merchants this week. She visits shops chosen at random; each visit signs the merchant with probability $p = 0.48$, independently. A visit that ends in a refusal takes 1.25 hours; a visit that ends in a signing takes 4.25 hours, because the contract and terminal installation are done on the spot. Let Y be the number of shops visited until the 3rd merchant signs, and T the total hours spent.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) $E(T) =$ _____ hours

(d) The standard deviation of T is _____ hours

Problem 15

seed 20260925

An importer must clear two shipments through the electronic customs portal, one after the other: she submits the declaration for shipment 1 until it is accepted, then starts on shipment 2. Each submission is accepted with probability $p = 0.40$ and otherwise bounced back for a technical correction; bounces happen independently from submission to submission. Let Y_1 and Y_2 be the numbers of submissions needed for the two shipments, and $T = Y_1 + Y_2$ the total number of submissions.

Give every answer to at least four significant figures.

(a) $P(T = 8) =$ _____

(b) $V(T) =$ _____

(c) $P(T \leq 5) =$ _____

Problem 16

seed 20260925

A research team studying tax compliance interviews randomly selected shop owners and asks: “In the last year, have you recorded any cash sale below its true value to reduce VAT?” Suppose a fraction 0.14 of owners have done so, and that 0.80 of **those** owners deny it. The remaining 0.86 of owners have not, and all of them truthfully answer no. Let Y be the number of interviews up to and including the first “yes”.

Give every answer to at least four significant figures.

(a) The probability p that a single interview produces a “yes”, $p =$ _____

(b) $P(Y = 3) =$ _____

(c) $E(Y) =$ _____

(d) The probability that 19 interviews produce no “yes” at all, _____

Problem 17

seed 20260925

A factoring company has bought an overdue invoice from a Baku construction supplier. Each week, independently, the debtor pays in full with probability $p = 0.16$. Let Y be the number of the week in which payment arrives. The factor’s cost of carrying the invoice is

$$C = 190Y + 15Y^2 \text{ AZN,}$$

where the quadratic term reflects provisioning rules that bite harder the longer an invoice stays unpaid.

Give every answer to at least four significant figures.

(a) $E[Y(Y - 1)] =$ _____

(b) $V(Y) =$ _____ (compute it from (a); do not just quote it)

(c) $E(C) =$ _____ AZN

(d) An analyst estimates the expected cost by plugging $E(Y)$ into the formula, $190 E(Y) + 15 [E(Y)]^2$. By how much does this **understate** $E(C)$? _____ AZN

Problem 18

seed 20260925

A bank's corporate sales team pitches its payroll service to firms taken at random from among its existing loan customers. Each pitch, independently, persuades the firm to move its payroll account with probability $p = 0.72$. The team's quarterly target is 3 new payroll clients. Let Y be the number of the pitch on which the 3rd client signs, and $p(y) = P(Y = y)$.

Give probabilities and ratios to at least four significant figures.

(a) $p(6)/p(5) =$ _____

(b) The most likely value of Y (the value y with the largest $p(y)$) is _____

(c) $P(Y = y)$ at that most likely value is _____

Problem 19

seed 20260925

An exploration consortium holds a licence block on the Caspian shelf. Each exploratory well costs 38 million AZN and, independently of the others, finds a commercial deposit with probability $p = 0.18$. A discovery is worth 317 million AZN (net present value of development, before exploration costs). The consortium drills wells one at a time and stops at the first discovery, but its board has approved at most 5 wells. Let N be the number of wells actually drilled, so $N = \min(Y, 5)$ where Y is geometric.

Give every answer to at least four significant figures.

(a) The probability that the programme finds a deposit, _____

(b) $E(N) =$ _____

(c) The expected net value of the programme (value of any discovery minus all drilling costs), _____ million AZN. (Enter a negative number if it is negative.)

(d) Decision:

- ☐ Launch the programme: its expected net value is positive
- ☐ Do not launch: its expected net value is negative

Problem 20

seed 20260925

An investment bank is building the order book for a Baku Stock Exchange IPO and needs 3 anchor investors. It approaches institutional funds one at a time; each fund, independently, commits with probability $p = 0.48$. Let Y be the number of the approach on which the 3rd commitment arrives, and let X_n be the number of commitments among the first n approaches.

Give every answer to at least four significant figures.

