

Week 7, Problem Set 1

Wackerly 3.7-3.8 - The Hypergeometric and Poisson Distributions; Midterm I Review

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A Central Bank examiner visits a branch that holds 16 mortgage files. Unknown to the examiner, 7 of them have a documentation error. The examiner pulls 4 files at random, all at once, so that every set of 4 files is equally likely to be chosen. Let Y be the number of files with an error in the examiner's sample.

Give every probability to at least four significant figures (enter a decimal number).

(a) $P(Y = 0) =$ _____

(b) $P(Y = 1) =$ _____

(c) $P(Y \geq 2) =$ _____

Problem 2

seed 20260925

A regional equity index lists 24 stocks, 12 of which are banks. A new fund picks 6 different index stocks at random, every set of 6 being equally likely. Let Y be the number of bank stocks the fund picks.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) The standard deviation of Y is _____

(d) The factor by which $V(Y)$ is smaller than the variance of a binomial random variable with $n = 6$ and $p = r/N$: $V(Y)/[np(1 - p)] =$ _____

Problem 3

seed 20260925

Calls to the card-services line of a Baku bank arrive at random, and the number Y of calls in a five-minute interval has a Poisson distribution with mean $\lambda = 2.20$.

Give every probability to at least four significant figures.

(a) $P(Y = 0) =$ _____

(b) $P(Y = 6) =$ _____

(c) $P(Y \leq 2) =$ _____

(d) $P(Y > 6) =$ _____

Problem 4

seed 20260925

A bank's card-fraud system raises alerts at random; the number Y of alerts on a given day has a Poisson distribution with mean $\lambda = 3.0$. Monitoring costs 380 AZN a day plus 30 AZN for every alert investigated, so the daily cost is $C = 380 + 30Y$ AZN.

Give every answer to at least four significant figures.

(a) $V(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $E(C) =$ _____ AZN

(d) The standard deviation of C is _____ AZN

Problem 5

seed 20260925

A microfinance lender holds 140 small loans to unrelated market traders. Each loan defaults this year with probability $p = 0.019$, independently of the others, so the number Y of defaults is binomial with $n = 140$ and $p = 0.019$.

Give every answer to at least four significant figures (enter decimal numbers).

(a) The parameter of the approximating Poisson distribution: $\lambda =$ _____

(b) The Poisson approximation to $P(Y = 0)$: _____

(c) The Poisson approximation to $P(Y \leq 2)$: _____

(d) The exact binomial value of $P(Y = 0)$: _____

Problem 6

seed 20260925

Customers walk into a small branch of a bank in Ganja according to a Poisson process with an average of 10 customers per hour.

Give every answer to at least four significant figures.

(a) The teller steps away for 8 minutes. The expected number of customers who arrive in that time is _____

(b) The probability that at least one customer arrives while the teller is away is _____

(c) The probability that exactly two customers arrive in a 15-minute window is _____

Problem 7

seed 20260925

A payments company buys card terminals in lots of 18. Testing a terminal is slow, so the company tests 5 terminals chosen at random from each lot (without replacement) and rejects the lot if more than one tested terminal is faulty; a rejected lot goes back to the supplier.

A particular lot contains 6 faulty terminals. Let Y be the number of faulty terminals among the 5 tested.

Give every answer to at least four significant figures.

(a) The probability that this lot is rejected is _____

(b) $E(Y) =$ _____

(c) $V(Y) =$ _____

Problem 8

seed 20260925

A district tax office has a register of 15 small firms, 6 of which under-report their turnover. It has capacity for 4 audits this year.

Each week the office draws one firm at random from the whole register, and a firm that has already been audited stays in the register and can be drawn again. This is repeated for 4 weeks.

Let Y be the number of audits that land on an under-reporting firm.

(a) Which model describes Y exactly?

- ☐ Hypergeometric with $N = 15$, $r = 6$, $n = 4$
- ☐ Binomial with $n = 4$, $p = 6/15$
- ☐ Poisson with mean $4 \times 6/15$
- ☐ Geometric with $p = 6/15$

Give the remaining answers to at least four significant figures.

(b) $P(Y = 1) =$ _____

(c) $V(Y) =$ _____

Problem 9

seed 20260925

A bank receives a batch of 19 newly personalised payment cards. Exactly 5 of them have a faulty chip, but nobody knows which. Before the batch is mailed to customers, the card centre tests n cards chosen at random, without replacement. Let Y be the number of faulty cards among those tested.

The centre wants the probability of catching at least one faulty card to be at least 0.70.

(a) The smallest sample size n that achieves this is _____

Give the probabilities to at least four significant figures.

(b) With that n , $P(Y \geq 1) =$ _____

(c) With one card fewer, $P(Y \geq 1) =$ _____

Problem 10

seed 20260925

A bank's data centre has 14 identical-looking servers in stock; 5 of them shipped with outdated firmware. Tonight the team takes 4 servers at random from stock and installs them. A server with current firmware takes 40 minutes to install; one with outdated firmware must be re-flashed first and takes 130 minutes. Let Y be the number of outdated servers among the 4, and T the total installation time in minutes.

Give every answer to at least four significant figures.

(a) The probability that none of the 4 servers needs re-flashing is _____

(b) $E(T) =$ _____ minutes

(c) $V(T) =$ _____ squared minutes

(d) The standard deviation of T is _____ minutes

Problem 11

seed 20260925

The number Y of claims per week on a small portfolio of cargo-insurance policies at a Baku insurer is modelled as Poisson with an unknown mean λ . Over many years, a fraction 0.120 of weeks have had no claims at all, and the actuary sets $P(Y = 0) = 0.120$. Weeks are treated as disjoint intervals of a Poisson process.

Give every answer to at least four significant figures.

(a) $\lambda =$ _____

(b) In a week in which at least one claim arrives, the probability that at least 4 arrive: $P(Y \geq 4 \mid Y \geq 1) =$ _____

(c) The probability of exactly two claims in total over the next 3 weeks is _____

Problem 12

seed 20260925

A payment switch in Baku receives card payments through two independent channels. In a given 10-millisecond interval, the number Y_1 of payments arriving through the mobile app is Poisson with mean 1.6, and the number Y_2 arriving through shop terminals is Poisson with mean 2.8. Treat the two counts as independent, and let $W = Y_1 + Y_2$.

Give every answer to at least four significant figures.

(a) $P(W = 3) =$ _____

(b) $V(W) =$ _____

(c) Given that $W = 3$, the probability that every one of the 3 payments came through the app: $P(Y_1 = 3 \mid W = 3) =$ _____

Problem 13

seed 20260925

A catalytic cracking unit at a refinery near Baku breaks down at random, and the number Y of breakdowns in a day is Poisson with mean $\lambda = 1.4$. Each breakdown costs more than the one before (the unit must be restarted from a colder state), and the day's revenue in thousand AZN is modelled as

$$R = 19000 - 250 Y^2.$$

Give every answer to at least four significant figures.

(a) $E(Y^2) =$ _____

(b) $E(R) =$ _____ thousand AZN

(c) The probability that the day's revenue is below 18000 thousand AZN is _____

seed 20260925

Problem 14

The security team of a bank logs cyber incidents, which occur according to a Poisson process with a long-run average of 2.8 incidents per month. Over the last 4 months, 15 incidents were logged. Let Y be the number of incidents in a 4-month period if the long-run rate still holds.

Give numerical answers to at least four significant figures.

(a) $E(Y) =$ _____

(b) The upper end of the interval $\mu \pm 2\sigma$: $\mu + 2\sigma =$ _____

(c) Is the observed count of 15 beyond $\mu + 2\sigma$?

☐ Yes: it lies more than two standard deviations above the mean

☐ No: it lies within two standard deviations of the mean

(d) The exact probability $P(Y \geq 15) =$ _____

seed 20260925

Problem 15

An agent selling life-insurance policies by phone makes calls one after another. Each call ends in a sale with probability $p = 0.14$, independently of every other call.

Give every answer to at least four significant figures.

(a) The probability that the agent's first sale comes on call number 6: _____

(b) The variance of the number of the call on which the first sale comes: _____

The agent's daily target is 2 sales.

(c) The probability that the second sale comes on call number 8: _____

(d) The expected number of calls needed to reach 2 sales: _____

(e) The standard deviation of the number of calls needed to reach 2 sales: _____

seed 20260925

Problem 16

A leasing company has 38 car leases outstanding to unrelated clients. Each lease goes into arrears this quarter with probability 0.08, independently of the others. Y is the number of leases that go into arrears this quarter.

(a) Which distribution does Y have?

- ☐ Binomial
- ☐ Geometric
- ☐ Negative binomial
- ☐ Hypergeometric
- ☐ Poisson

Give the remaining answers to at least four significant figures.

(b) $E(Y) =$ _____

(c) $V(Y) =$ _____

Problem 17

seed 20260925

A leasing company in Sumgait holds 26 equipment leases to unrelated small firms. Each lease defaults this year with probability $p = 0.075$, independently of the others. Each lease has an exposure of 11 thousand AZN, and the company recovers part of it on default, losing a fraction 0.65 of the exposure. Let Y be the number of defaults and L the total credit loss in thousand AZN.

Give every answer to at least four significant figures (enter decimal numbers).

(a) $E(L) =$ _____ thousand AZN

(b) The standard deviation of L is _____ thousand AZN

(c) The exact probability that the loss exceeds 14.30 thousand AZN: _____

(d) The Poisson approximation to the probability in (c): _____

Problem 18

seed 20260925

An electronics shop in Baku is clearing an old tablet model with a list price of 300 AZN. For every unit sold during the day, the shop multiplies the price of the next unit by 0.90. So if Y units are sold during the day, the price displayed at closing time is $300(0.90)^Y$ AZN.

Give every answer to at least four significant figures.

(a) Suppose the number of buyers Y is Poisson with mean 2.0. The expected closing price is _____ AZN

(b) The closing price computed at the mean number of buyers, $300(0.90)^{2.0}$, is _____ AZN

(c) Suppose instead that exactly 10 customers will see the offer and each buys one unit with probability 0.30, independently of the others. The expected closing price is then AZN

Problem 19

seed 20260925

A customs officer at the Baku port has a batch of 14 import declarations, of which 5 understate the value of the goods. She checks the declarations one at a time in a random order (every ordering equally likely), never checking the same one twice.

Give every answer to at least four significant figures.

(a) The probability that the **second** understated declaration she finds is the one checked in position 5:

(b) The probability that at least two of the first 5 declarations she checks are understated:

(c) The expected number of understated declarations among the first 5 checked:

Problem 20

seed 20260925

The compliance desk of a bank flags large-value transfers for review. The number Y of transfers flagged in a day is modelled as Poisson with an unknown mean λ . From its records the desk estimates

$$E[Y(Y - 1)] = 15.3664.$$

Give numerical answers to at least four significant figures.

(a) $\lambda =$ _____

(b) $E(Y^2) =$

(c) The most likely number of flagged transfers in a day (the value y with the largest $p(y)$) is

(d) The probability of that most likely number:

