

Week 8, Problem Set 1

Wackerly 3.9 - Moments and Moment-Generating Functions

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A retail brokerage in Baku records Y , the number of trades a randomly chosen active client places on a given day. From last quarter's data,

y	0	1	2	3
$p(y)$	0.47	0.24	0.24	0.05

(a) Using Definition 3.14, write the moment-generating function of Y as a function of t . (Type e^{2t} as $e^{(2*t)}$, for example.)

$m(t) =$ _____

(b) Use Theorem 3.12 to find $E(Y) = m^{(1)}(0) =$ _____

(c) Use Theorem 3.12 to find $E(Y^2) = m^{(2)}(0) =$ _____

Problem 2

seed 20260925

A dealer quotes a benchmark Azerbaijani government bond. Let Y be the change in its price from one day's close to the next, measured in ticks. The dealer's model is

y	-1	0	1	2
$p(y)$	0.13	0.46	0.33	0.08

Use the book's notation: $\mu'_k = E(Y^k)$ is the k th moment about the origin (Definition 3.12) and $\mu_k = E[(Y - \mu)^k]$ the k th central moment (Definition 3.13). Give answers to at least four significant figures.

(a) $\mu'_1 =$ _____

(b) $\mu'_2 =$ _____

(c) $\mu'_3 =$ _____

(d) $\mu_2 =$ _____

(e) $\mu_3 =$ _____

Problem 3

seed 20260925

The operational-risk unit of a Baku bank tracks a weekly count Y (failed card transactions, disputed transfers, and so on) and, in its internal model, summarises Y only by its moment-generating function

$$m(t) = (0.40e^t + 0.60)^{19}.$$

(a) Which distribution does Y have?

- ☐ Binomial
☐ Geometric
☐ Poisson
☐ Hypergeometric

(b) By inspection, $E(Y) =$ _____

(c) By inspection, $V(Y) =$ _____

Give numerical answers to at least four significant figures.

Problem 4

seed 20260925

A bank's call centre phones 14 pre-screened customers with a credit-card offer. Each customer accepts independently of the others, and Y , the number who accept, has moment-generating function

$$m(t) = (0.55e^t + 0.45)^{14}.$$

Differentiate $m(t)$ and use Theorem 3.12. Give answers to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $V(Y) =$ _____

Problem 5

seed 20260925

Let Y be the number of business days it takes a cross-border payment sent through a Baku correspondent bank to settle. The bank's treasury summarises Y by its moment-generating function

$$m(t) = 0.24e^t + 0.33e^{2t} + 0.11e^{3t} + 0.32e^{4t}.$$

Give answers to at least four significant figures.

(a) $P(Y = 3) =$ _____

(b) $P(Y \geq 3) =$ _____

(c) $E(Y) =$ _____

(d) $V(Y) =$ _____

Problem 6

seed 20260925

A brokerage's risk system models Y , the number of margin calls a leveraged client receives in a month. Its documentation does not give $p(y)$; it gives only the first terms of the power-series expansion of the moment-generating function about $t = 0$:

$$m(t) = 1 + 1.11t + 1.17105t^2 + 1.028988t^3 + \dots$$

Give answers to at least four significant figures.

(a) $\mu'_1 = E(Y) =$ _____

(b) $\mu'_2 = E(Y^2) =$ _____

(c) $V(Y) =$ _____

(d) $\mu'_3 = E(Y^3) =$ _____

(e) Each margin call costs the client a 40 AZN handling fee. Expected monthly margin-call fees:

_____ AZN

Problem 7

seed 20260925

A Baku dealer makes a market in a thinly traded corporate bond. The number of trades Y it executes in the bond on a given day has moment-generating function

$$m(t) = e^{1.60(e^t - 1)}.$$

Give answers to at least four significant figures.

(a) Use Theorem 3.12 to find the third moment about the origin, $\mu'_3 = E(Y^3) =$ _____

(b) Find the third central moment $\mu_3 = E[(Y - \mu)^3] =$ _____

(c) The dealer earns a flat 9 AZN commission per trade, so its daily commission revenue is $R = 9Y$. Find the third central moment of R , $E[(R - E(R))^3] =$ _____

Problem 8

seed 20260925

A proprietary trader places 8 short-horizon bets on unrelated events. Each bet wins with probability 0.66, independently of the others. A winning bet gains 2 and a losing bet loses 4, both in hundreds of AZN. Let Y be the number of winning bets, so Y is binomial with $m_Y(t) = (0.66e^t + 0.34)^8$, and let X be the trader's total P&L (hundreds of AZN).

(a) Express X as $X = aY + b$ and use it to write the moment-generating function of X . (Type e^{2t} as $e^{(2*t)}$.)

$m_X(t) =$ _____

(b) $E(X) =$ _____ (hundreds of AZN)

(c) $V(X) =$ _____ (squared hundreds of AZN)

Give numerical answers to at least four significant figures.

Problem 9

seed 20260925

A motor insurer in Baku notices that many of its policies cover cars that are hardly ever driven. Its actuaries model the number of claims Y on a randomly chosen policy in a year through the moment-generating function

$$m(t) = 0.20 + 0.80 e^{1.40(e^t - 1)}.$$

Give answers to at least four significant figures.

(a) $P(Y = 0) =$ _____

(b) $E(Y) =$ _____

(c) $E(Y^2) =$ _____

(d) $V(Y) =$ _____

Problem 10

seed 20260925

An agent for a Baku life insurer phones prospects one at a time. Calls are independent and each ends in a sale with the same probability. Let Y be the number of the call on which the agent makes the first sale of the day. The insurer's model gives Y the moment-generating function

$$m(t) = \frac{0.14e^t}{1 - 0.86e^t}.$$

Give answers to at least four significant figures.

(a) Use Theorem 3.12: $E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $V(Y) =$ _____

(d) Each call costs the insurer 12 AZN in the agent's time and phone charges. Expected cost of the calls up to and including the first sale: _____ AZN

(e) $P(Y \leq 3) =$ _____

Problem 11

seed 20260925

A bank has 10 branches. Each branch meets its monthly deposit-growth target with probability 0.84, independently of the others, so the number Y of branches that meet it is binomial with

$$m_Y(t) = (0.84e^t + 0.16)^{10} \text{ and mean } \mu = 8.40.$$

(a) Write the moment-generating function of $X = Y - \mu$, the deviation of the count from its mean. (Type e^{2t} as $e^{(2*t)}$.)

$m_X(t) =$ _____

(b) Theorem 3.12 applied to m_X gives the central moments of Y . Find $\mu_2 = E[(Y - \mu)^2] =$ _____

(c) $\mu_3 = E[(Y - \mu)^3] =$ _____

(d) Which describes the distribution of Y ?

- ☐ Longer right tail (third central moment positive)
- ☐ Longer left tail (third central moment negative)
- ☐ Symmetric about the mean (third central moment zero)

Give numerical answers to at least four significant figures.

Problem 12

seed 20260925

A fintech start-up's fraud model produces a count Y each hour. The model's documentation gives only the moment-generating function, written by a developer who did not simplify it:

$$m(t) = \frac{(5e^t + 2)^{11}}{7^{11}}.$$

(a) Which distribution does Y have?

- ☐ Binomial
- ☐ Geometric
- ☐ Poisson
- ☐ None of these

(b) $E(Y) =$ _____

(c) $V(Y) =$ _____

Give numerical answers to at least four significant figures.

Problem 13

seed 20260925

A mortgage broker in Baku submits 10 applications this week. A model the broker's analyst fitted says the number Y of those applications that are approved has moment-generating function

$$m(t) = (0.76e^t + 0.24)^{10}.$$

Give every probability to at least four significant figures (enter decimals).

(a) $P(Y = 7) =$ _____

(b) $P(Y \leq 7) =$ _____

Problem 14

seed 20260925

The card-fraud desk of a Baku bank receives Y automated alerts per hour during the evening shift. From a year of logs, the desk models Y with moment-generating function

$$m(t) = e^{4.38(e^t - 1)}.$$

Give answers to at least four significant figures.

(a) $\mu = E(Y) =$ _____

(b) $\sigma =$ _____

(c) The desk staffs for “normal” hours, those with $|Y - \mu| \leq 2\sigma$. Find $P(|Y - \mu| \leq 2\sigma) =$ _____

seed 20260925

Problem 15

A corporate treasury in Baku holds a one-week currency position. Its P&L X , in thousands of US dollars, is modelled as

x	-4	0	6
$p(x)$	0.20	0.37	0.43

(Type e^{2t} as $e^{(2*t)}$.)

(a) $m_X(t) =$ _____

(b) Use Theorem 3.12: $E(X) =$ _____ (thousand USD)

(c) $V(X) =$ _____

(d) The treasury reports in manat, at 1.7 AZN per USD, so the P&L in thousands of AZN is $W = 1.7X$. Write the moment-generating function of W .

$m_W(t) =$ _____

(e) $V(W) =$ _____

Give numerical answers to at least four significant figures.

seed 20260925

Problem 16

A small Baku business applies for a working-capital loan at one bank after another, stopping at the first approval. Decisions are independent and each bank approves with the same probability. The number Y of the application on which the first approval comes has moment-generating function

$$m_Y(t) = \frac{0.22e^t}{1 - 0.78e^t}.$$

Let $X = Y - 1$ be the number of **rejections** before the first approval.

(a) Write the moment-generating function of X and simplify it.

$m_X(t) =$ _____

(b) Use Theorem 3.12 on m_X : $E(X) =$ _____

(c) $V(X) =$ _____

(d) Every rejected application costs the business a non-refundable 75 AZN appraisal fee. Expected total of such fees: _____ AZN

(e) $P(X \geq 3) =$ _____

Give numerical answers to at least four significant figures.

Problem 17

seed 20260925

A market-maker's risk report gives only the moment-generating function of its P&L X over one trading session, in hundreds of AZN:

$$m_X(t) = e^{-20t} (0.45e^{2t} + 0.55)^{15}.$$

Give answers to at least four significant figures.

(a) $E(X) =$ _____

(b) $V(X) =$ _____

(c) $P(X > 0) =$ _____

(Hint: find a random variable Y you recognise and constants with $X = aY + b$.)

Problem 18

seed 20260925

A Baku bank buys protection on a corporate borrower. The borrower survives each year independently with probability 0.96 and defaults with probability 0.04, so the year Y in which it first defaults ($Y = 1, 2, 3, \dots$) is geometric with $p = 0.04$. If default happens, the protection seller pays 5 million AZN at the end of year Y . With continuously compounded interest at $r = 0.115$, the value today of that payment is $5D$, where $D = e^{-rY}$ is the discount factor.

(a) Starting from Definition 3.14, derive the moment-generating function of Y .

$m(t) =$ _____

(b) Explain to yourself why $E(D) = m(-r)$, then compute $E(D) =$ _____

(c) Expected present value of the protection payment: _____ million AZN

(d) $V(D) =$ _____

Give numerical answers to at least four significant figures.

Problem 19

seed 20260925

A share listed on the Baku Stock Exchange trades at 32 AZN today. An analyst models next month's price as

$$S = 32(0.93)^Y,$$

where Y , the number of negative news shocks in the month, is Poisson with $m_Y(t) = e^{3.8(e^t - 1)}$, and each shock knocks 7% off the price. (Nothing else moves the price in this stylised model.)

