Week 9, Problem Set 2

Wackerly 4.3-4.4 - Expected Values for Continuous Random Variables; the Uniform Distribution

> This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A state bus operator in Baku tenders a one-year diesel-supply contract. From past tenders, the procurement office models the lowest bid Y (in thousands of AZN) as uniformly distributed between $\theta_1 = 195$ and $\theta_2 = 255$.

Give every answer to at least four significant figures.

(a) The probability that the lowest bid on the next tender is below 207.00 thousand AZN: _____

(b) The probability that the lowest bid exceeds 246.00 thousand AZN: _____

(c) $E(Y) =$ _____ thousand AZN

(d) $V(Y) =$ _____ (thousand AZN)^2

Problem 2

seed 20260925

The daily change Y in the price of a short-term Central Bank note, measured in qapik per 100 AZN of face value, is modelled by the density

$$f(y) = \begin{cases} k, & -34 \le y \le 34, \\ 0, & \text{elsewhere.} \end{cases}$$

Give every number to at least four significant figures.

(a) $k =$ _____

(b) For $-34 \le y \le 34$, the distribution function is $F(y) =$ _____ (enter a formula in y)

(c) $F(13) = P(Y \le 13) =$ _____

(d) The probability that the price rises by more than 7 qapik in a day: _____

(e) $V(Y) =$ _____ (qapik)^2

Problem 3

seed 20260925

Let Y be the largest single cash withdrawal (in hundreds of thousands of AZN) from the vault of a regional bank branch on a given day. The branch's liquidity model uses the density

$$f(y) = \begin{cases} \dfrac{3y^2}{3.4^3}, & 0 \le y \le 3.4, \\ 0, & \text{elsewhere.} \end{cases}$$

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $V(Y) =$ _____

(d) $P(Y \le 2.890) =$ _____

Problem 4

seed 20260925

Large block trades in a bank's shares arrive on the Baku Stock Exchange according to a Poisson distribution. The trading session lasts 240 minutes, and on one particular day exactly one block trade was reported. Let Y be the time (in minutes after the open) at which it took place.

Give every probability to at least four significant figures.

(a) The probability that the block trade took place during the last 60 minutes of the session: _____

(b) The probability that it took place between minute 55 and minute 100 of the session: _____

(c) $E(Y) =$ _____ minutes

Problem 5

seed 20260925

Let Y be the proportion of a day during which a bank's ATM network is fully operational. The operations team fits the density

$$f(y) = \begin{cases} 4y^3, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The network's daily contribution to profit (in thousands of AZN) is $X = 115Y - 13$.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____

(c) $E(X) =$ _____ thousand AZN

(d) $V(X) =$ _____ (thousand AZN)^2

Problem 6

seed 20260925

A chain of utility-payment kiosks in Baku does not give small change: every bill is rounded **up** to the next multiple of 5 qapik, and the kiosk keeps the difference. Let Y be the amount (in qapik) kept on one bill. Because the qapik part of a bill is effectively arbitrary, Y is modelled as uniformly distributed on $(0, 5)$.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ qapik

(b) $V(Y) =$ _____ (qapik)^2

(c) The probability that more than 4.5 qapik is kept on a bill: _____

(d) A kiosk processes 16500 bills a month. The expected total amount it keeps in a month, in AZN (100 qapik = 1 AZN): _____

Problem 7

seed 20260925

A bank's SME loans that default are pursued through a workout unit. For a defaulted loan with exposure 190 thousand AZN, the proportion Y of the exposure that is lost (the loss given default) has density

$$f(y) = \begin{cases} 0.50\,y + 2.25\,y^2, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The net amount the bank recovers, after a workout cost of 3.00 thousand AZN, is $W = 190(1 - Y) - 3.00$ thousand AZN.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____

(b) $E(Y^2) =$ _____

(c) $V(Y) =$ _____

(d) $E(W) =$ _____ thousand AZN

(e) $V(W) =$ _____ (thousand AZN)^2

Problem 8

seed 20260925

A wholesaler sells to retailers on 34-day payment terms. Let Y be the number of days after delivery at which a retailer pays. Most retailers pay soon and few wait until the deadline, and the credit department models

$$f(y) = \begin{cases} \dfrac{2}{34}\left(1 - \dfrac{y}{34}\right), & 0 \le y \le 34, \\ 0, & \text{elsewhere.} \end{cases}$$

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ days

(b) $E(Y^2) =$ _____

(c) $V(Y) =$ _____

(d) The probability that a retailer takes more than 25.50 days to pay: _____

Problem 9

seed 20260925

The proportion Y of the business day during which every teller at a busy bank branch is serving a customer has density

$$f(y) = \begin{cases} c\,y^{1}(1-y)^{5}, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The branch prices the cost of customers queueing as $C = 360 + 800\,Y$ AZN per day.

You may use, for non-negative integers r and s,

$$\int_0^1 y^r (1-y)^s \, dy = \frac{r!\,s!}{(r+s+1)!}.$$

Give every answer to at least four significant figures.

(a) $c =$ _____

(b) $E(Y) =$ _____

(c) $V(Y) =$ _____

(d) $E(C) =$ _____ AZN

Problem 10

seed 20260925

A bank runs its overnight risk calculations on rented cloud servers. The number Y of server-hours used in a week has density

$$f(y) = \begin{cases} \dfrac{12}{28^{4}}\, y^{2}(28-y), & 0 \le y \le 28, \\ 0, & \text{elsewhere.} \end{cases}$$

Server time costs 29 AZN per hour, so the weekly bill is $C = 29Y$ AZN.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ hours

(b) $V(Y) =$ _____ hours^2

(c) $E(C) =$ _____ AZN

(d) $V(C) =$ _____ AZN^2

(e) The IT budget allows 649.60 AZN a week. The probability that the weekly bill exceeds the budget:

_____

Problem 11

seed 20260925

When the cash-dispensing module of one of a bank's ATMs fails, a replacement is ordered from the supplier. The delivery time Y (in days) is uniformly distributed between 1 and 7 days. The module costs 850 AZN, and the fees and customer goodwill lost while the machine is down grow faster than linearly with the delay, so the total cost of one failure is

$$C = 850 + 50\,Y^2 \quad \text{AZN}.$$

Give every answer to at least four significant figures.

(a) The probability that delivery takes longer than 4.00 days: _____

(b) $E(Y) =$ _____ days

(c) $E(Y^2) =$ _____

(d) $E(C) =$ _____ AZN

Problem 12

seed 20260925

A payment processor submits card transactions to the clearing house in batches. The time Y (in minutes) from submission until a batch is settled is uniformly distributed between 35 and 75 minutes.

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ minutes

(b) $V(Y) =$ _____ minutes^2

(c) The probability that a batch takes more than 69.00 minutes to settle: _____

(d) A batch submitted 45.00 minutes ago has still not settled. The probability that it will take more than 69.00 minutes in total: _____

(e) For the batch in (d), the expected total settlement time: _____ minutes

Problem 13

seed 20260925

An insurer's property desk records claims above 500,000 AZN, which arrive according to a Poisson distribution. During one 70-day reporting period, exactly one such claim was reported. Let Y be the day (measured continuously from the start of the period) on which it was reported.

Give every answer to at least four significant figures.

(a) The probability that the claim came in during the first 19 days: _____

(b) The probability that it came in between day 30 and day 44: _____

(c) An auditor who has seen only the first 30 days of the file knows that no large claim arrived in them. Given that information, the probability that the claim came in between day 30 and day 44: _____

(d) Given the information in (c), the expected day on which the claim was reported: _____

Problem 14

seed 20260925

The Central Bank's deposit-auction platform opens at 10:00 and accepts bids for 15 minutes, then closes for 30 minutes while it clears them, then opens again for 15 minutes, and so on. A commercial bank's treasury system sends its bid at a time Y that is uniformly distributed over the first 105 minutes after 10:00. A bid that arrives while the platform is closed is queued and entered the moment the platform reopens.

Give every answer to at least four significant figures.

(a) The probability that the bid is accepted on arrival (the platform is open): _____

(b) Given that the bid arrives while the platform is closed, its expected queueing time: _____ minutes

(c) The (unconditional) expected queueing time of the bid: _____ minutes

Problem 15

seed 20260925

Among the borrowers of a consumer-credit company who miss an instalment's due date, the delay Y (in days) until they pay is uniformly distributed on $(0, 16)$. The company's penalty G (in AZN) depends on the delay through a grace period of 8 days:

$$G = g(Y) = \begin{cases} 0, & 0 < Y \le 8, \\ 11 + 4.5\,(Y - 8), & 8 < Y < 16. \end{cases}$$

Give every answer to at least four significant figures.

(a) The probability that a late borrower is charged a penalty: _____

(b) $E(G) =$ _____ AZN

(c) The expected penalty of a borrower who **is** charged one: _____ AZN

Problem 16

seed 20260925

The monthly return X (in percent) of a manat money-market fund is modelled by the density

$$f(x) = \begin{cases} 0.16, & -1 < x \le 0, \\ 0.16 + 1.36\,x, & 0 < x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

An investor holds 470 thousand AZN in the fund, so her gain for the month is $G = 470X/100$ thousand AZN.

Give every answer to at least four significant figures.

(a) $P(X > 0) =$ _____

(b) $E(X) =$ _____

(c) $E(X^2) =$ _____

(d) $V(X) =$ _____

(e) $E(G) =$ _____ thousand AZN

(f) The standard deviation of G: _____ thousand AZN

Problem 17

seed 20260925

A crude-oil tanker is due at the Sangachal terminal some time in a 9-hour window, and its arrival time Y (hours after the window opens) is uniformly distributed on $(0, 9)$. The terminal must schedule its berth crew to start at a fixed time t in the window.

- If the tanker arrives before t, it waits, and the terminal pays demurrage of 850 AZN per hour of waiting.
- If it arrives after t, the crew stands idle at a cost of 450 AZN per hour.

So the cost is $C_t = 850(t - Y)$ if $Y < t$ and $C_t = 450(Y - t)$ if $Y \geq t$.

Give every answer to at least four significant figures.

(a) The terminal currently schedules the crew for $t = 7.2$. Its expected cost $E(C_t)$ is _____ AZN

(b) The start time t^* that minimises the expected cost: _____ hours

(c) The minimum expected cost $E(C_{t^*})$: _____ AZN

(d) The expected cost if the crew is scheduled for the **mean** arrival time $t = E(Y)$: _____ AZN

Problem 18

seed 20260925

An analyst models the price Y (in AZN) of a Baku-listed bank share on the expiry date of a one-month option as uniformly distributed between 24 and 53 AZN. A European call option with strike $K = 37.05$ AZN pays $C = \max(Y - 37.05, 0)$ at expiry; the put with the same strike pays $P = \max(37.05 - Y, 0)$.

Give every answer to at least four significant figures.

(a) The probability that the call finishes in the money ($Y > K$): _____

(b) $E(C) =$ _____ AZN

(c) $V(C) =$ _____ AZN^2

(d) $E(P) =$ _____ AZN

Problem 19

seed 20260925

Defaults in a pool of corporate loans held by a Baku bank occur according to a Poisson distribution. Over the next 5 years the bank's stress scenario assumes that **exactly one** default happens, at time Y (in years), with a loss of 2.2 million AZN at that moment. Discounting continuously at rate $\delta = 0.145$ per year, the present value of the loss is

$$W = 2.2\,e^{-0.145\,Y} \quad \text{million AZN}.$$

Give every answer to at least four significant figures.

(a) $E(Y) =$ _____ years

(b) $E(W) =$ _____ million AZN

(c) The present value obtained by discounting the loss from the **mean** default time, $2.2\,e^{-0.145\,E(Y)}$:
_____ million AZN

(d) $V(W) =$ _____ (million AZN)^2

Problem 20

seed 20260925

An insurer's actuaries model the size Y (in thousands of AZN) of a commercial fire claim by its distribution function

$$F(y) = \begin{cases} 0, & y < 0, \\ 1 - \left(1 - \dfrac{y}{190}\right)^2, & 0 \le y \le 190, \\ 1, & y > 190. \end{cases}$$

The insurer buys reinsurance that pays the part of each claim above a retention of 57.00 thousand AZN, that is, $R = \max(Y - 57.00,\, 0)$.

For a non-negative continuous Y you may use $E(Y) = \int_0^\infty [1 - F(y)]\,dy$ (Exercise 4.34). Give every number to at least four significant figures.

(a) For $0 \le y \le 190$, the density is $f(y) =$ _____ (enter a formula in y)

(b) $E(Y) =$ _____

(c) $E(Y^2) =$ _____

(d) $V(Y) =$ _____

(e) $E(R) =$ _____ thousand AZN