

Week 10, Problem Set 1

Wackerly 4.5 - The Normal Probability Distribution

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A bank's credit-scoring model reports every applicant's score in **standardised** form, so that the standardised score Z of a randomly chosen applicant is a standard normal random variable ($\mu = 0$, $\sigma = 1$).

Use Table 4, Appendix 3, and give each probability to four decimal places.

(a) $P(0 \leq Z \leq 0.73) =$ _____

(b) The bank flags applicants whose score is unusually high for a manual fraud check: $P(Z > 2.34) =$

(c) $P(-0.79 \leq Z \leq 1.97) =$ _____

(d) $P(-1.93 \leq Z \leq -0.58) =$ _____

Problem 2

seed 20260925

The monthly electricity bill Y of a household in one Baku district is approximately normally distributed with mean $\mu = 42$ AZN and standard deviation $\sigma = 12$ AZN.

Use Table 4 and give each answer to four decimal places.

(a) What fraction of households pay between 30 and 61.8 AZN?

(b) The distribution company sends an energy-saving letter to every household whose bill exceeds 62.4 AZN. What fraction of households receive the letter?

(c) What fraction of households pay less than 30 AZN?

Problem 3

seed 20260925

An energy analyst models the change Y (USD per barrel) in the Brent crude price over the coming month by the density

$$f(y) = \frac{1}{8\sqrt{2\pi}} e^{-(y+1.1)^2/128}, \quad -\infty < y < \infty.$$

(a) $E(Y) =$ _____

(b) $V(Y) =$ _____ and the standard deviation of Y is _____

(c) The median of Y , the value $\phi_{0.5}$ with $P(Y \leq \phi_{0.5}) = 0.5$, is _____

(d) Using Table 4, find $P(Y > 7.54)$, the probability that the monthly change exceeds 7.54 USD per barrel, to four decimal places.

Problem 4

seed 20260925

A bank standardises the scores its credit model produces, so that the standardised score Z of a randomly chosen applicant has a standard normal distribution. The risk team sets its cut-offs directly on the Z scale.

Use Table 4. Table 4 lists z in steps of 0.01; an answer from either neighbouring table entry is accepted.

(a) A premium credit card is offered only to the highest-scoring applicants, a fraction 0.2119 of the whole pool. Find the cut-off z_0 with $P(Z > z_0) = 0.2119$.

$z_0 =$ _____

(b) Find z_1 such that $P(Z < z_1) = 0.9535$.

$z_1 =$ _____

(c) The “standard” band is the symmetric interval that contains the middle 0.90 of all scores. Find $z_2 > 0$ with $P(-z_2 < Z < z_2) = 0.90$.

$z_2 =$ _____

Problem 5

seed 20260925

The weekly amount a bank branch spends on IT maintenance and repairs has been observed over a long period to be approximately normally distributed with mean 900 AZN and standard deviation 120 AZN. Next week’s budget is 1050 AZN.

Use Table 4 and give each probability to four decimal places.

(a) What is the probability that next week’s costs exceed the budget?

(b) What is the probability that next week’s costs are below 678 AZN?

(c) Assume the costs in different weeks are independent and the budget stays at 1050 AZN each week. What is the probability that the budget is exceeded in none of the next 4 weeks?

Problem 6

seed 20260925

The monthly growth rate Y of retail deposits at a commercial bank, in percent, is approximately normal with mean $\mu = 0.6$ and standard deviation $\sigma = 1.9$.

(a) The empirical rule of Chapter 1 says that about 95% of a mound-shaped distribution lies within 2 standard deviation(s) of the mean. Give that interval $(\mu - 2\sigma, \mu + 2\sigma)$ for Y .

lower end = _____ %, upper end = _____ %

Now use Table 4 and give each probability to four decimal places.

(b) The exact normal probability that Y falls in the interval of (a):

(c) $P(-5.1 < Y < 4.4) =$ _____

(d) The probability that deposits grow by more than 5.35 percent in a month:

Problem 7

seed 20260925

An insurance company holds a securities portfolio worth 70 million AZN. Its risk department models the portfolio's daily return R , in percent, as normal with mean $\mu = 0.01$ and standard deviation $\sigma = 2.25$.

The one-day Value at Risk at confidence level 99.5% is the loss that is exceeded on only 0.005 of days: if r^* is the 0.005 quantile of R , that is $P(R \leq r^*) = 0.005$, then $\text{VaR} = -r^* \times (\text{portfolio value})/100$.

Use Table 4; for quantiles an answer from either neighbouring table entry is accepted.

(a) Find the quantile r^* (in percent).

$r^* =$ _____ %

(b) Find the one-day 99.5% VaR in million AZN.

VaR = _____ million AZN

(c) What is the probability that tomorrow's loss exceeds 2,906,750 AZN? Give it to four decimal places.

(d) If the model is right, on how many of the next 250 trading days should the loss exceed the VaR of (b), on average?

Problem 8

seed 20260925

The total motor-insurance claims Y that an insurer pays in a week are approximately normal with mean 360 thousand AZN and standard deviation 58 thousand AZN.

Use Table 4. Answers from either neighbouring table entry are accepted.

(a) The insurer keeps a weekly liquidity reserve r and wants the claims to exceed it in only a fraction **0.1** of weeks, $P(Y > r) = 0.1$. How large must r be?

$r =$ _____ thousand AZN

(b) Find the lower quartile $\phi_{0.25}$ of weekly claims, the value with $P(Y \leq \phi_{0.25}) = 0.25$.

$\phi_{0.25} =$ _____ thousand AZN

(c) Find the interquartile range $\phi_{0.75} - \phi_{0.25}$.

IQR = _____ thousand AZN

Problem 9

seed 20260925

At a filling station, the amount Y of fuel actually dispensed when a customer pays for 20 litres is normally distributed with mean μ , which the operator sets by calibrating the pump, and standard deviation $\sigma = 0.38$ litres.

Use Table 4. Answers from either neighbouring table entry are accepted.

(a) The consumer-protection inspectorate requires that customers be short-changed ($Y < 20$) on at most 1% of fills. What is the smallest setting μ that meets the rule?

$\mu =$ _____ litres

(b) With μ set as in (a), the pump gives away $\mu - 20$ litres per fill on average. The station makes 1100 such fills a day and fuel costs 1.00 AZN per litre. What is the expected daily cost of the overfill?

_____ AZN

(c) A new pump lets the operator adjust σ . What is the largest σ for which the amount dispensed lies within 0.54 litres of the mean with probability at least 0.99?

$\sigma =$ _____ litres

Problem 10

seed 20260925

Two credit bureaus report scores on different scales. Among all adults in the country, bureau A's scores are approximately normal with mean 635 and standard deviation 100; bureau B's are approximately normal with mean 490 and standard deviation 120.

(a) A bank requires a bureau A score of at least 655. What fraction of adults score below 655 on bureau A? (Table 4, four decimal places.)

(b) For applicants who only have a bureau B score, the bank wants a **comparable** standard: a cut-off that excludes the same fraction of adults on bureau B. What cut-off should it use?

(c) Applicant X has a bureau A score of 805; applicant Y has a bureau B score of 754. Give each applicant's z -score within their own bureau, to two decimal places.

X: $z =$ _____ Y: $z =$ _____

(d) Relative to their own population, who has the stronger credit score?

- ☐ Applicant X (bureau A)
- ☐ Applicant Y (bureau B)
- ☐ They are equally strong

Problem 11

seed 20260925

A pension saver compares two funds. The annual return of the bond fund, R_A , is approximately normal with mean 3.6% and standard deviation 4%. The annual return of the equity fund, R_B , is approximately normal with mean 18.75% and standard deviation 25%.

Use Table 4 and give probabilities to four decimal places.

(a) Probability that each fund loses money over a year:

$P(R_A < 0) =$ _____ $P(R_B < 0) =$ _____

(b) The saver's target is a return above 8%:

$P(R_A > 8) =$ _____ $P(R_B > 8) =$ _____

(c) Which fund is less likely to lose money?

- ☐ The bond fund
- ☐ The equity fund

(d) Which fund is more likely to beat the 8% target?

- ☐ The bond fund
- ☐ The equity fund

Problem 12

seed 20260925

An oil-trading company in Baku sells 20 thousand barrels of Brent crude forward, for delivery in three months at the fixed price $F = 80$ USD per barrel. Its analysts model the Brent spot price Y at delivery as normal with mean $\mu = 74$ and standard deviation $\sigma = 10$ USD per barrel. The profit and loss of the position, in USD, is

$$W = 20,000 (F - Y) = -20,000 Y + 20,000 F.$$

A linear function $aY + b$ of a normal random variable is itself normal (this is proved in Chapter 6).

(a) $E(W) =$ _____ USD

(b) The standard deviation of W is _____ USD

Use Table 4 for (c) and (d), four decimal places.

(c) $P(W > 0)$, the probability that the forward sale makes a profit:

(d) The probability that the position loses more than 240,000 USD:

Problem 13

seed 20260925

The credit scores S of applicants for a consumer loan at a bank are approximately normal with mean 620 and standard deviation 100. An applicant is approved if $S > 630$, and approved applicants with $S > 765$ are offered the bank's preferential interest rate.

Use Table 4; give probabilities to four decimal places.

(a) What proportion of applicants are approved?

(b) An applicant is known to have been approved. What is the probability that they are offered the preferential rate?

(c) Management considers replacing the rule by “offer the preferential rate to the top 15% of all applicants”. What score cut-off would that be? (An answer from either neighbouring table entry is accepted.)

Problem 14

seed 20260925

A mobile operator finds that the monthly data use Y of its prepaid subscribers is approximately normal with mean 15.5 GB and standard deviation 5.8 GB. It uses quantiles of this distribution to design its tariffs.

Use Table 4. Answers from either neighbouring table entry are accepted.

(a) Find the data allowance C such that $P(Y < C) = 0.8643$.

$C =$ _____ GB

(b) The heaviest 10% of users are to be offered an unlimited plan. Above what monthly use does a subscriber fall into that group?

_____ GB

(c) Find the interval, symmetric about the mean, that contains the middle 70% of subscribers' data use.

from _____ GB to _____ GB

Problem 15

seed 20260925

The daily return R (in percent) of a government Eurobond held by an Azerbaijani pension fund is approximately normal with mean 0.02 and standard deviation 0.85. The fund's risk report calls a day "bad" if $R < -1.1615$. Treat returns on different days as independent.

Use Table 4 and give answers to four decimal places.

(a) The probability that a given day is bad:

(b) The probability that none of the next 12 trading days is bad:

(c) The probability that at least two of the next 12 trading days are bad:

(d) The expected number of bad days among the next 12:

Problem 16

seed 20260925

A mint strikes coins whose nominal weight is 3.90 g. A coin is accepted only if its weight is in the interval 3.90 ± 0.0722 g, that is between 3.8278 and 3.9722 g; the rest are melted down. The weight of a coin from the current press setting is normal with mean $\mu = 3.8791$ g and standard deviation $\sigma = 0.038$ g.

Use Table 4; probabilities to four decimal places.

(a) With the current setting, what fraction of coins is rejected?

(b) σ cannot be changed, but μ can. Which mean weight minimises the fraction rejected?

$\mu =$ _____ g

(c) What fraction is rejected at that setting?

(d) The mint strikes 750,000 coins a day. About how many fewer coins a day are rejected after the adjustment?

Problem 17

seed 20260925

A bank's analysts believe that the monthly card spending Y (AZN) of its premium cardholders is approximately normal, but they do not know μ or σ . What they do know from the card system is that a fraction **0.1020** of cardholders spend less than 557.4 AZN a month and a fraction **0.0322** spend more than 1743 AZN.

Use Table 4. Answers consistent with the neighbouring table entries are accepted.

(a) Find μ and σ .

$\mu =$ _____ AZN $\sigma =$ _____ AZN

(b) What fraction of premium cardholders spend more than 1211 AZN a month? (Four decimal places.)

(c) The bank sets each card's monthly limit at the 99th percentile of spending. What is that limit?

_____ AZN

Problem 18

seed 20260925

A bank's treasury runs a foreign-currency position of 20 million AZN. The position's daily return R (in percent) is modelled as normal with mean 0.04 and standard deviation 0.70. The daily loss is $L = -R \times 20/100$ million AZN. The board has set a limit: the one-day 99% Value at Risk, the loss exceeded with probability only 0.01, must not be more than 378,400 AZN.

Use Table 4; quantile answers consistent with either neighbouring table entry are accepted.

(a) Compute the one-day 99% VaR, in AZN.

_____ AZN

(b) Is the limit breached?

- ☐ Yes: the 99% VaR is larger than the limit
☐ No: the 99% VaR is within the limit

(c) What is the probability that tomorrow's loss exceeds 378,400 AZN? (Four decimal places.)

(d) Keeping the position at 20 million AZN and the mean at 0.04%, what is the largest daily standard deviation (in %) compatible with the limit?

_____ %

(e) Keeping $\sigma = 0.70$, what is the largest position (million AZN) compatible with the limit?

Problem 19

seed 20260925

A risk analyst has 252 months of monthly returns of a broad government-bond index. She standardises each return using the sample mean and standard deviation, and counts the months on which the return was more than 2.95 standard deviations away from the mean, in either direction. She finds 2 of them.

Use Table 4.

(a) If the returns were normal, what is the probability that a single month's return lies more than 2.95 standard deviations from the mean? (Four decimal places.)

(b) How many such months would the normal model lead her to expect in the sample?

(c) What should she conclude?

- ☐ The count is close to what the normal model predicts; on this evidence there is no reason to doubt normality.
- ☐ There are several times more extreme returns than the normal model predicts: the tails are heavier than normal, so normality is doubtful.
- ☐ There are fewer extreme returns than predicted, so the returns must follow a uniform distribution.

(d) Table 4 also lists areas far in the tail. Under a normal model, a fall of more than 4.5 standard deviations below the mean happens on average once in how many months? (Enter $1/P(Z > 4.5)$; answers within 2% are accepted.)

Problem 20

seed 20260925

A bank's scoring model gives every loan applicant a score S . From past data, applicants who later default have scores that are normal with mean 595 and standard deviation 40; applicants who repay have scores that are normal with mean 659 and the same standard deviation. A fraction $\pi = 0.14$ of applicants would default if given a loan.

The bank approves every applicant whose score exceeds a cut-off c , and chooses c so that only a fraction 0.0099 of would-be defaulters are approved.

Use Table 4; give probabilities to four decimal places.

(a) Find the cut-off c . (An answer from either neighbouring table entry is accepted.)

$c =$ _____

(b) What fraction of applicants who would repay are approved?

(c) What fraction of all applicants are approved?

(d) An applicant has just been approved. What is the probability that they will default?