

Week 14, Problem Set 2

Wackerly 5.9 - The Multinomial Probability Distribution

This is one randomly generated version of the set. Every student receives different numbers, so the values here will not match your own copy — follow the method, not the arithmetic.

Problem 1

seed 20260925

A trader at a Baku brokerage who hedges a jeweller's gold purchases classifies each daily London close of the gold price as **up** (more than 0.5% above the previous close), **flat** (within 0.5%) or **down** (more than 0.5% below). She models the days as independent, with

$$P(\text{up}) = 0.44, \quad P(\text{flat}) = 0.25, \quad P(\text{down}) = 0.31$$

on every day. Over the next $n = 7$ trading days let Y_1, Y_2, Y_3 be the numbers of up, flat and down days.

Give each answer as a decimal to at least four significant figures.

(a) $P(Y_1 = 4, Y_2 = 1, Y_3 = 2) =$ _____

(b) The expected number of up days, $E(Y_1) =$ _____

(c) The variance of the number of down days, $V(Y_3) =$ _____

Problem 2

seed 20260925

A mobile operator's shop in Ganja signs up $n = 11$ new prepaid subscribers on a typical day. Each chooses one of three tariffs, independently of the others, with probabilities

$$\text{Mini: } p_1 = 0.39, \quad \text{Standard: } p_2 = 0.44, \quad \text{Unlimited: } p_3 = 0.17.$$

Let Y_1, Y_2, Y_3 be the numbers of the day's subscribers who choose Mini, Standard and Unlimited. Give each answer to at least four significant figures.

(a) $E(Y_1) =$ _____

(b) $V(Y_2) =$ _____

(c) The standard deviation of Y_3 is _____

(d) $\text{Cov}(Y_1, Y_3) =$ _____

Problem 3

seed 20260925

A bank's corporate book contains $n = 9$ firms rated BBB at the start of the year. From the rating agency's transition matrix, each firm independently ends the year upgraded (to A or better) with probability **0.14**, still BBB with probability **0.77**, or downgraded (to BB or worse) with probability **0.09**. Let Y_1, Y_2, Y_3 be the numbers of firms upgraded, unchanged and downgraded.

Give each number to at least four significant figures.

(a) $\text{Cov}(Y_1, Y_3) =$ _____

(b) $\text{Cov}(Y_1, Y_2) =$ _____

(c) The correlation coefficient ρ between Y_1 and Y_3 is _____

(d) Which pair of counts has the most negative covariance?

- ☐ Upgrades and downgrades
- ☐ Upgrades and firms staying BBB
- ☐ Downgrades and firms staying BBB

Problem 4

seed 20260925

An insurer in Baku covers a delivery company's fleet of $n = 9$ vans under separate motor policies. Over a year each policy, independently, ends up in one of three classes: **claim-free** (probability **0.74**), **small claims only** (probability **0.16**), or **at least one large claim** (probability **0.10**). Let Y_1, Y_2, Y_3 be the numbers of policies in the three classes.

Give each answer as a decimal to at least four significant figures.

(a) The probability that no policy has a large claim, $P(Y_3 = 0) =$ _____

(b) $P(Y_2 = 3) =$ _____

(c) $P(Y_3 \geq 2) =$ _____

(d) The expected number of claim-free policies, $E(Y_1) =$ _____

Problem 5

seed 20260925

A microfinance organisation in Lankaran disburses $n = 8$ new loans this month. Three months later each loan, independently, is in one of four states:

- current, with probability $p_1 = 0.72$;
- 1 to 30 days late, with probability $p_2 = 0.17$;
- 31 to 90 days late, with probability $p_3 = 0.06$;
- in default, with probability $p_4 = 0.05$.

Let Y_1, Y_2, Y_3, Y_4 be the numbers of loans in the four states. Give each answer to at least four significant figures.

(a) The probability that exactly **2** loans are late (in either lateness bucket) and exactly **1** are in default, i.e. $P(Y_2 + Y_3 = 2, Y_4 = 1) =$ _____

(b) $P(Y_2 + Y_3 = 2) =$ _____

(c) $E(Y_2 + Y_3) =$ _____

Problem 6

seed 20260925

A coffee chain in Baku records how customers pay. For a randomly chosen customer the probabilities are

Method	Cash	Bank card	Mobile wallet	Loyalty points
Probability	0.36	0.39	0.14	0.11

Customers choose independently. The next $n = 6$ customers are observed and Y_1, Y_2, Y_3, Y_4 count the payments by cash, card, wallet and points.

Give each answer as a decimal to at least four significant figures.

(a) $P(Y_1 = 1, Y_2 = 2, Y_3 = 2, Y_4 = 1) =$ _____

(b) The probability that none of the 6 customers pays with the mobile wallet is _____

Problem 7

seed 20260925

A commercial bank's branch network opens $n = 28$ new deposit accounts in a week. Each new customer, independently, chooses a demand deposit (probability 0.39), a 6-month term deposit (probability 0.39) or a 12-month term deposit (probability 0.22). Let Y_1, Y_2, Y_3 be the numbers of accounts of each kind.

The treasury watches $D = Y_2 - Y_3$, the excess of 6-month over 12-month accounts, because it shifts the maturity profile of the bank's funding.

Give each answer to at least four significant figures.

(a) $\text{Cov}(Y_2, Y_3) =$ _____

(b) $E(D) =$ _____

(c) $V(D) =$ _____

(d) The standard deviation of D is _____

Problem 8

seed 20260925

A bank's collections unit receives $n = 26$ newly overdue credit-card accounts each week. Independently, each account needs a reminder phone call (probability 0.54), needs a field visit by an agent (probability 0.13), or is paid by the customer without any contact (probability 0.33). Let Y_1 and Y_2 be the numbers of accounts needing a call and a visit.

A call costs 6 AZN and a visit costs 28 AZN, so the week's collection cost is $C = 6Y_1 + 28Y_2$ AZN.

Give each answer to at least four significant figures.

(a) $E(C) =$ _____ AZN

(b) $V(C) =$ _____ AZN²

(c) The standard deviation of C is _____ AZN

Problem 9

seed 20260925

A credit fund holds bonds of $n = 11$ issuers rated BB. Over a year each issuer, independently, is upgraded to investment grade (probability **0.17**), stays BB (probability **0.68**) or is downgraded to B or worse (probability **0.15**). Let Y_1, Y_2, Y_3 count the upgraded, unchanged and downgraded issuers.

At year end the risk report arrives first with only the downgrades: $Y_3 = 4$. Given this information, answer the following to at least four significant figures.

(a) For one of the issuers **not** downgraded, the conditional probability that it was upgraded is _____

(b) $P(Y_1 = 1 \mid Y_3 = 4) =$ _____

(c) $E(Y_1 \mid Y_3 = 4) =$ _____

(d) $V(Y_1 \mid Y_3 = 4) =$ _____

Problem 10

seed 20260925

A retail client of a Baku broker places $n = 11$ limit orders on a foreign stock through the broker's app during a week. Each order, independently, ends up as an executed buy (probability **0.54**), an executed sell (probability **0.30**) or cancelled before execution (probability **0.16**). Let Y_1, Y_2, Y_3 be the numbers of executed buys, executed sells and cancellations.

Give each answer as a decimal to at least four significant figures.

(a) The probability of at least **3** cancellations, $P(Y_3 \geq 3) =$ _____

(b) $P(Y_1 = 5, Y_3 = 0) =$ _____

(c) $P(Y_1 = 5 \mid Y_3 = 0) =$ _____

Problem 11

seed 20260925

A currency-exchange kiosk at Baku's Heydar Aliyev airport serves customers who each, independently, buy US dollars (probability **0.44**), sell US dollars (probability **0.28**) or make a transaction in another currency (probability **0.28**). Among the next $n = 4$ customers let Y_1, Y_2, Y_3 be the numbers in each group. The cashier's dollar float falls if more customers buy dollars than sell them.

Give each answer as a decimal to at least four significant figures.

(a) $P(Y_1 > Y_2) =$ _____

(b) $P(Y_1 = Y_2) =$ _____

Problem 12

seed 20260925

A mortgage-backed bond issued by a Baku bank is backed by a pool of $n = 56$ residential mortgages. Over the coming year each mortgage, independently, is prepaid in full (probability **0.14**), defaults (probability **0.04**) or stays current (probability **0.82**). Let Y_1, Y_2, Y_3 be the counts, and let $N = Y_1 + Y_2$ be the number of mortgages that leave the pool, which is what shortens the bond's life.

Give each answer to at least four significant figures.

(a) $E(N) =$ _____

(b) $\text{Cov}(Y_1, Y_2) =$ _____

(c) $V(N) =$ _____

(d) The correlation coefficient between Y_1 and Y_2 is _____

Problem 13

seed 20260925

A consultancy surveys $n = 240$ randomly chosen Baku households and asks which bank is their main bank. Suppose the true shares are $p_1 = 0.41$ for Bank A, $p_2 = 0.20$ for Bank B and $p_3 = 0.39$ for all other banks together. Treat the answers as independent. Let Y_1, Y_2, Y_3 be the numbers of households naming Bank A, Bank B and another bank.

Bank A's marketing team will quote the estimated lead $\hat{L} = \frac{Y_1}{n} - \frac{Y_2}{n}$.

Give each answer to at least four significant figures.

(a) $E(\hat{L}) =$ _____

(b) $V(\hat{L}) =$ _____

(c) The standard deviation of $\hat{L}$ is _____

Problem 14

seed 20260925

A development bank funds a cohort of $n = 11$ small-business loans with the same term. At maturity each loan, independently, has been repaid early (probability 0.29), repaid on schedule (0.49), restructured (0.11) or has defaulted (0.11). Let Y_1, Y_2, Y_3, Y_4 count the four outcomes.

The workout department reports first: exactly $Y_4 = 2$ loans defaulted. Given this, answer the following to at least four significant figures.

(a) $P(Y_1 = 3, Y_2 = 4, Y_3 = 2 \mid Y_4 = 2) =$ _____

(b) $E(Y_3 \mid Y_4 = 2) =$ _____

Problem 15

seed 20260925

A telecom operator's call centre handles n contract renewals a day. Each customer, independently, picks a premium bundle, a standard bundle or a basic bundle, with fixed probabilities p_1, p_2, p_3 . The numbers Y_1, Y_2, Y_3 choosing each bundle are therefore multinomial. The analytics team does not report n or the p_i . It reports only

$$E(Y_1) = 10.24, \quad V(Y_1) = 6.9632, \quad E(Y_2) = 12.16.$$

Give each answer to at least four significant figures.

(a) $p_1 =$ _____

(b) $n =$ _____

(c) $p_2 =$ _____

(d) $\text{Cov}(Y_1, Y_2) =$ _____

Problem 16

seed 20260925

A risk desk classifies each daily move of the Brent crude price as **up** (a rise of more than 1%, probability **0.44**), **down** (a fall of more than 1%, probability **0.21**) or **quiet** (within 1%, probability **0.35**). Days are modelled as independent. Over the next $n = 8$ trading days let Y_1, Y_2, Y_3 be the numbers of up, down and quiet days.

Give each answer as a decimal to at least four significant figures.

(a) The probability of exactly 5 up days and at most one down day, $P(Y_1 = 5, Y_2 \leq 1) =$ _____

(b) $P(Y_2 \leq 1) =$ _____

(c) $P(Y_1 = 5 \mid Y_2 \leq 1) =$ _____

Problem 17

seed 20260925

An FX dealer at a Baku bank answers $n = 56$ requests for quotes from corporate clients on a typical day. Each request, independently, ends in one of four ways:

	small ticket	mid ticket	large informed ticket	quote not taken
probability	0.44	0.17	0.05	0.34
dealer's P\&L (AZN)	+54	+141	-310	0

Let Y_1, Y_2, Y_3, Y_4 be the day's counts, so the day's P&L is $P = 54Y_1 + 141Y_2 - 310Y_3$.

Give each answer to at least four significant figures.

(a) $E(P) =$ _____ AZN

(b) In Theorem 5.12, the variance of P is the sum of the three $a_i^2 V(Y_i)$ terms plus the covariance terms $2 \sum_{i < j} a_i a_j \text{Cov}(Y_i, Y_j)$. The covariance terms add up to _____ AZN²

(c) $V(P) =$ _____ AZN²

(d) The standard deviation of the day's P&L is _____ AZN

Problem 18

seed 20260925

A bank holds $n = 11$ corporate loans of 1 million AZN each, all to firms rated BBB today. From the rating agency's one-year transition matrix, each firm independently ends the year in one of four states. The bank's year-end charge per loan (capital held, or the loss if the firm defaults) depends on that state:

year-end state	A	BBB	BB	default
probability	0.08	0.84	0.06	0.02
charge (thousand AZN)	4	10	19	570

Let Y_1, Y_2, Y_3, Y_4 be the numbers of firms in the four states and $K = 4Y_1 + 10Y_2 + 19Y_3 + 570Y_4$ the total charge, in thousand AZN.

Give each answer to at least four significant figures.

(a) $E(K) =$ _____

(b) $V(K) =$ _____

(c) The standard deviation of K is _____

(d) The probability that no firm defaults and at most one is downgraded to BB, $P(Y_4 = 0, Y_3 \leq 1) =$ _____

Problem 19

seed 20260925

A junior trader on a Baku bank's treasury desk opens and closes $n = 6$ positions in a week. Each round trip, independently, closes at a profit (probability **0.49**), at a loss (probability **0.28**) or flat (probability **0.23**). Let Y_1, Y_2, Y_3 be the numbers of winning, losing and flat trades. The desk head watches the net win count $D = Y_1 - Y_2$.

Give each answer as a decimal to at least four significant figures.

(a) The probability that the week has as many losing as winning trades, $P(Y_1 = Y_2) =$ _____

(b) $E(Y_1 Y_2) =$ _____

(c) $V(D) =$ _____

(d) $E(D^2) =$ _____

Problem 20

seed 20260925

A quant at an Azerbaijani asset manager prices a digital option on a stock index by simulation. In each of $n = 80$ independent scenarios the index finishes in one of three regions:

- above the strike, so the option pays off (probability $p_1 = 0.28$);
- below its starting level (probability $p_2 = 0.40$);
- between the start and the strike (probability $p_3 = 0.32$).

Let Y_1, Y_2, Y_3 be the numbers of scenarios in each region. The quant needs np_1 . She knows p_2 exactly from a closed-form formula, so she can use Y_2 as a **control variate**:

$$U = Y_1 + c(Y_2 - np_2),$$

which has $E(U) = np_1$ for every constant c .

Give each answer to at least four significant figures.

(a) $\text{Cov}(Y_1, Y_2) =$ _____

