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Problem 1

Samantha owns 6 different mathematics books and 4 different computer science books and wish to fill 5 positions on a shelf. If the first 2 positions are to be occupied by math books and the last 3 by computer science books, in how many ways can this be done? ________

Correct Answers:

- 720

+6pc-1pc
Problem 2

A girl owns 4 pairs of pants, 1 shirts, 7 ties, and 6 jackets. How many different outfits can the girl wear to school if each outfit must consist of one of each item?

There are ________ different outfits.

Solution: By the generalized basic principle of counting, there are $4 * 1 * 7 * 6 = 168$ different outfits possible.

Correct Answers:

- 168

+6pc-1pc
Problem 3

A standard Missouri state license plate consists of a sequence of two letters, one digit, one letter, and one digit. How many such license plates can be made?

____

A standard New York state license plate consists of a sequence of three letters followed by three digits. How many such license plates can be made?

____

Correct Answers:

- 1.7576×10^6
- 1.7576×10^7

+6pc-1pc

Problem 4

A fair 6-sided die is rolled 4 times and the resulting sequence of 4 numbers is recorded.

How many different sequences are possible? ________

How many different sequences consist entirely of even numbers? ________

How many different sequences are possible if the first, third, and fourth numbers must be the same?

____

Correct Answers:

- 1296
- 81
- 36

+6pc-1pc
Problem 5

A park bench can seat 4 people. How many seating arrangements are possible if 4 people out of a group of 14 want to sit on the park bench?

____

Correct Answers:

- 24024

+6pc-1pc
Problem 6

You have a penny, a nickel, a dime, a toonie, and a loonie. How many different (non-zero) sums of money can you produce?

___

Solution:

There are two possibilities for each of the five coins: being included or not. For each possibility for a single coin, there are two possibilities for each of the other four. Multiplying, the total number of possibilities is therefore $2^5 = 32$. However, this contains the case when no coin is included, so excluding this possibility leaves 31 different sums of money.
Alternatively, note that we can choose five, four, three, two, or one coin(s), and the number of combinations in each case can be added together to give
$1 + \binom{5}{4} + \binom{5}{3} + \binom{5}{2} + 5 = 31$.

Correct Answers:

- 31

+6pc-1pc

Problem 7

Our Indiscrete Mathematics course has

- 20 students from the the College of Arts, 13 of whom are female;
- 28 students from the the College of Engineering and Informatics, 9 of whom are female;
- 30 students from the the College of Science, 10 of whom are female.

How many ways can we choose a single class rep?

Answer: ________

How many ways can we choose **three reps**, one from each of the three Colleges?

Answer: ________

How many ways can we choose **three reps**, one from each of the three Colleges, so that **exactly one** is female?

Answer: ________

Correct Answers:

- $30 + 28 + 20$
- $30 \cdot 28 \cdot 20$
- $10 \cdot 19 \cdot 7 + 20 \cdot 9 \cdot 7 + 20 \cdot 19 \cdot 13$

+6pc-1pc

Problem 8

At a restaurant there are 5 kinds of pasta and 2 ttypes of sauce.
How many different ways can a customer order one kind of pasta and one type of sauce?

- A. 10
- B. 5
- C. 7
- D. 20

Solution:

Solution

The customer may first choose his pasta in 5 ways. For each of these choices there are 2 sauce choices so the customer has $5 \times 2 = 10$ ways.

Correct Answers:

- A

+6pc-1pc

Problem 9

Standard automobile license plates in a country display 2 numbers, followed by 3 letters, followed by 2 numbers. How many different standard plates are possible in this system? (Assume repetition of letters and numbers is allowed.)
Your answer is : ___________

Solution: There are 26 choices for each of the letters and 10 choices for each of the numbers. Since repetition is allowed, there are a total of $10^2 \cdot 26^3 \cdot 10^2 = 10^4 \cdot 26^3 = 175760000$ possible standard license plates.

Correct Answers:

- 175760000

+6pc-1pc
Problem 10

A man has five ties, six shirts, and five different pairs of trousers. How many different ways does he have to dress himself?

___
Solution:

There are five choices for both the tie and the trousers, and six for the shirt. By the fundamental rule of combinatorics there must be
$5 \times 6 \times 5 = 150$
possible ways to combine them.

Correct Answers:

- 150

+6pc-1pc
Problem 11

Evaluate

$$\frac{25!}{23!}$$

$\frac{25!}{23!} =$ ________________
Correct Answers:

- 600

+6pc-1pc
Problem 12

Simplify the expression

$$\frac{(4n+4)!}{(4n-3)!}.$$

$\frac{(4n+4)!}{(4n-3)!} =$ ________________
Correct Answers:

- $(4n-3+1)\,(4n+(-1))\,(4n+0)\,(4n+1)\,(4n+2)\,(4n+3)\,(4n+4)$

+6pc-1pc
Problem 13

How many different 8-letter permutations can be formed from 6 identical H's and two identical T's?

Answer: ________

Correct Answers:

- $\dfrac{8!}{2\cdot(6!)}$

+6pc-1pc
Problem 14

In a cohort of thirty graduating students, there are three different prizes to be awarded. If no student can receive more than one prize, in how many different ways could the prizes be awarded?

___
Solution:

The prizes are different, so we have permutations to consider. Since the number of ways of permuting n different objects taken k at a time is
$\frac{n!}{(n-k)!}$
we have
$\frac{30!}{(30-3)!} = \frac{30!}{27!} = 24360$
different ways.

Correct Answers:

- 24360

+6pc-1pc
Problem 15

In how many ways can 3 students be seated in a row of 3 chairs if Jack insists on sitting in the first chair?
Your answer is : ________

Correct Answers:

- 2

+6pc-1pc
Problem 16

In how many ways can 4 different novels, 2 different mathematics books, and 1 biology book be arranged on a bookshelf if

(a) the books can be arranged in any order?

Answer: ________

(b) the mathematics books must be together and the novels must be together?

Answer: ________

(c) the mathematics books must be together but the other books can be arranged in any order?

Answer: ________
Correct Answers:

- 5040
- 288
- 1440

+6pc-1pc
Problem 17

How many 4-element subsets containing the letter A can be formed from the set $\{A, B, C, D, E, F, G\}$?

Answer: ________

Correct Answers:

- 20

+6pc-1pc
Problem 18

A standard deck of cards consists of four suits (clubs, diamonds, hearts, and spades), with each suit containing 13 cards (ace, two through ten, jack, queen, and king) for a total of 52 cards in all.

How many 7-card hands will consist of exactly 2 kings and 3 queens?

____
Correct Answers:

- 22704

+6pc-1pc
Problem 19

A standard deck of cards consists of four suits (clubs, diamonds, hearts, and spades), with each suit containing 13 cards (ace, two through ten, jack, queen, and king) for a total of 52 cards in all.

How many 7-card hands will consist of exactly 3 hearts and 3 clubs?

____
Correct Answers:

- 2.1267×10^6

+6pc-1pc

Problem 20

In how many ways can a person invite 4 out of their 14 closest friends to a dinner party?

____

Correct Answers:

- 1001

+6pc-1pc
Problem 21

A company received a shipment of 31 laser printers, including 6 that are defective. 2 of these printers are selected to be used in the copy room.

(a) How many selections can be made? _________

(b) How many of these selections will contain no defective printers? _________
Correct Answers:

- 465
- 300

+6pc-1pc
Problem 22

A school dance committee is to consist of 2 freshmen, 3 sophomores, 4 juniors, and 5 seniors. If 5 freshmen, 7 sophomores, 9 juniors, and 7 seniors are eligible to be on the committee, in how many ways can the committee be chosen?
Your answer is : ___________

Solution: There are $\binom{5}{2}$ ways to choose 2 freshmen for the committee, $\binom{7}{3}$ ways to choose 3 sophomores for the committee, $\binom{9}{4}$ ways to choose 4 juniors for the committee, and $\binom{7}{5}$ ways to choose 5 seniors for the committee. So by the generalized basic principle of counting, there are a total of

$$\binom{5}{2} \cdot \binom{7}{3} \cdot \binom{9}{4} \cdot \binom{7}{5} = \frac{5!}{2!3!} \cdot \frac{7!}{3!4!} \cdot \frac{9!}{4!5!} \cdot \frac{7!}{5!2!} = 926100$$

different possible committees.

Correct Answers:

- 926100

+6pc-1pc
Problem 23

Suppose you are managing 18 employees, and you need to form three teams to work on different projects. Assume that all employees will work on a team, and that each employee has the same qualifications/skills so that everyone has the same probability of getting choosen. In how many different ways can the teams be chosen so that the number of employees on each project are as follows:

5, 4, 9

Answer: ____

Correct Answers:

- 6126120

+6pc-1pc

Problem 24

A secret code for a bank vault consists of 4 letters, then 4 digits and then 4 more letters.

How many different codes are possible?
Answer:________________

How many codes are possible if repeating letters and digits is not allowed?
Answer:________________

How many codes are possible if repeats are not allowed and the first letter must be 'G' and the second digit must be '1'?
Answer:________________

If the wrong code is entered the vault automatically locks and the alarm sounds. Suppose repeating letters and digits are allowed in the code. What is ther probability of a thief breaking into the vault if the thief has no prior knowledge of the secret code?
Answer:________________

Correct Answers:

- 2088270645760000
- 317474277120000
- 1221054912000
- 4.788651327501E-16

+6pc-1pc

Problem 25

Are the following statements true or false?

[?] 1. $\sum_{k=1}^{n} k(n+1-k) = \binom{n+3}{2}$

[?] 2. $\sum_{k=0}^{n} 3^k \binom{n}{k} = 2^n$

[?] 3. $D_n = (n-1)\big(D_{n-1} + D_{n-2}\big)$ where D_n is the number of derangements of n objects, and $n \geq 2$

[?] 4. $D_n = (n-1)\big(D_{n-1} - D_{n-2}\big)$ where D_n is the number of derangements of n objects, and $n \geq 2$

[?] 5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

[?] 6. $\binom{n}{k} = \binom{n}{n-k}$

[?] 7. $n! = (n-1)\big((n-1)! + (n-2)!\big) \quad$ for $n \geq 2$

Correct Answers:

- F
- F
- T
- F
- T
- T
- T

+6pc-1pc
Problem 26

A coin is tossed 12 times.
a) How many different outcomes are possible?

____
b) How many different outcomes have exactly 8 heads?

____
c) How many different outcomes have at least 2 heads ?

____
d) How many different outcomes have at most 8 heads?

____
Solution:

SOLUTION

a) There are 2 possibilities for each of the 12 flips, so there are a total of $2^{12} = 4096$ different possible outcomes.

b) Outcomes with exactly 8 heads can be completely described by indicating which 8 of the 12 tosses land on heads (since the others must be tails). Therefore there are $\binom{12}{8} = \frac{12!}{8!4!} = 495$ possible outcomes with exactly 8 heads.

c) First we count the outcomes that do not have at least 2 heads, i.e. the ones that have either 0 or 1 head. There is only one outcome with no heads (the outcome consisting of 12 tails) and there are $\binom{12}{1} = 12$ outcomes with exactly 1 head. As we found in part a), there are a total of 2^{12} possible outcomes overall, so the number of outcomes with at least 2 heads is $2^{12} - (12 + 1) = 4083$.

d) As in part c), we first count the outcomes that do not satisfy the given condition, i.e. the ones that have more than 8 heads. There is one outcome with exactly 12 heads. There are $\binom{12}{11}$ outcomes with exactly 11 heads, $\binom{12}{10}$ outcomes with exactly 10 heads, and $\binom{12}{9}$ outcomes with exactly 9 heads. So there are $2^{12} - (1 + \binom{12}{11} + \binom{12}{10} + \binom{12}{9}) = 3797$ outcomes with at most 8 heads.

Correct Answers:

- 4096
- 495
- 4083
- 3797

+6pc-1pc

Problem 27

$\frac{9!}{6!3!} =$ ________

Correct Answers:

- 84

+6pc-1pc

Problem 28

A company has 3415 employees. Each employee is to be given an ID number that consists of one letter followed by 3 digits.
How many different ID numbers are possible?: __________
Is it possible to give each employee a different ID number using this scheme?
Your answer is (input Yes or No): __________

Correct Answers:

- 26000
- Yes

+6pc-1pc

Problem 29

The annual National No Spying Day is celebrated at KAOS headquarters this year. There are 9 Control agents and 19 KAOS agents attending. How many ways can we choose a team of 5 agents if 3 team members need to be from Control and 2 from KAOS?

____

How many ways can we choose a team of 5 agents if at least 1 team member needs to be from Control?

____

Correct Answers:

- 14364
- 86652

+6pc-1pc

Problem 30

How many ways are there to seat 2 people in a row of 6 chairs? ________

Solution: Approach 1: We can think of first choosing 2 of the 6 chairs for people to sit in. There are $\binom{6}{2} = \frac{6!}{2!4!}$ possible choices. Then there are 2! possible ways for the 2 people to sit in the 2 chosen chairs. So there are a total of

$$\binom{6}{2} \cdot 2! = \frac{6!}{2!4!} \cdot 2! = \frac{6!}{4!} = 30$$

possible ways to seat 2 people in a row of 6 chairs.

Approach 2: We can think of first choosing one of the 6 chairs for the first person, then choosing one of the 5 remaining chairs for the second person, and so on. This again gives a total of $\frac{6!}{4!} = 30$ possible seating arrangements.

Correct Answers:

- 30

+6pc-1pc
Problem 31

How many ways are there to select 12 countries in the United Nations to serve on a council if 3 is selected from a block of 55, 2 are selected from a block of 61 and 7 are selected from the remaining 73 countries?

Correct Answers:

- 78225108329550496