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Problem 1

Suppose that the number of dice thrown is a binomial random variable with n = 13 and p = 0.89. What is the expected number of spots showing on the thrown dice?

Expected number of spots = ________

Correct Answers:

- 40.495

+6pc-1pc
Problem 2

A baseball player has a lifetime batting average of 0.258. If, in a season, this player has 220 ”at bats”, what is the probability he gets 58 or more hits?

Probability of 58 or more hits = ________
Correct Answers:

- 0.449759517787841

+6pc-1pc
Problem 3

Find the probability of throwing a sum of 7 at least 4 times in 9 throws of a pair of fair dice.

answer: ____

Correct Answers:

- 0.048021492214093

+6pc-1pc
Problem 4

Suppose the number of children in a household has a **binomial distribution** with parameters $n = 24$ and $p = 60\%$.

Find the probability of a household having:

(a) 17 or 23 children ____
(b) 21 or fewer children ____
(c) 18 or more children ____
(d) fewer than 23 children ____
(e) more than 21 children ____
Correct Answers:

- 0.0960595864123246
- 0.999338206073081

- 0.0959614713693864
- 0.999919447517249
- 0.000661793926918897

+6pc-1pc
Problem 5

Suppose that you flip a coin 12 times. What is the probability that you achieve at least 7 tails?

____
Correct Answers:

- 0.387207

+6pc-1pc
Problem 6

Suppose that you randomly draw one card from a standard deck of 52 cards. After writing down which card was drawn, you replace the card, and draw another card. You repeat this process until you have drawn 18 cards in all. What is the probability of drawing at least 8 hearts?

____

For the experiment above, let X denote the number of hearts that are drawn. For this random variable, find its expected value and standard deviation.

$E(X) =$ _________
$\sigma =$ _________
Correct Answers:

- 0.056948
- 4.5
- 1.83712

+6pc-1pc
Problem 7

It is known that a certain lacrosse goalie will successfully make a save 85.9% of the time. Suppose that the lacrosse goalie attempts to make 15 saves. What is the probability that the lacrosse goalie will make at least 12 saves?

____

Let X be the random variable which denotes the number of saves that are made by the lacrosse goalie. Find the expected value and standard deviation of the random variable.

$E(X) =$ _________
$\sigma =$ _________
Correct Answers:

- 0.849491
- 12.885
- 1.34788

+6pc-1pc
Problem 8

One interpretation of a baseball player's batting average is as the empirical probability of getting a hit each time the player goes to bat. If a player with a batting average of 0.264 bats 4 times in a game, and each at-bat is an independent event, what is the probability of the player getting at least on hit in the game?

____

Correct Answers:

- 0.706565

+6pc-1pc
Problem 9

A coin is tossed 10 times.

a) How many different outcomes are possible? _________

b) What is the probability of getting exactly 3 heads? _________

c) What is the probability of getting at least 2 heads? _________

d) What is the probability of getting at most 6 heads? _________

Correct Answers:

- 1024
- 0.1171875
- 0.9892578125
- 0.828125

+6pc-1pc
Problem 10

In a family with 7 children, excluding multiple births, what is the probability of having exactly 2 girls?
Assume that having a boy is as likely as having a girl at each birth.

____

Correct Answers:

- 0.164062

+6pc-1pc
Problem 11

Suppose that you roll two 6 sided dice.

a) What is the size of the sample space? _________

b) What is the probability that the sum of the dice is a 8? _________

c) What is the probability that the sum of the dice is at least a 8? ________

d) What is the probability that the sum of the dice is a prime number? ________

Correct Answers:

- 36
- 0.138889
- 0.416667
- 0.416667

+6pc-1pc

Problem 12

A biased coin that is more likely to land on tail than head is tossed 50 times. Which of the following statements must be correct?

- A. The random variable that represents the total number of heads tossed has a mean greater than 25.
- B. It is always the case that there are less than 25 heads tossed out of the 50 tosses.
- C. Both (A) and (B).
- D. Neither (A) nor (B).

Correct Answers:

- D

+6pc-1pc

Problem 13

A report says that 82% of British Columbians over the age of 25 are high school graduates. A survey of randomly selected residents of a certain city included 1290 who were over the age of 25, and 1012 of them were high school graduates.
Is the city's result of 1012 unusually high, low, or neither?
Answer: [Select one/High/Low/Neither]

Correct Answers:

- Low

+6pc-1pc

Problem 14

75% of the employees in a specialized department of a large software firm are computer science graduates. A project team is made up of 9 employees.

Part a) What is the probability to 3 decimal digits that all the project team members are computer science graduates? ___

Part b) What is the probability to 3 decimal digits that exactly 3 of the project team members are computer science graduates? ___

Part c) What is the most likely number of computer science graduates among the 9 project team members? Your answer should be an integer. If there are two possible answers, please select the smaller of the two integers. ___

Part d) There are 49 such projects running at the same time and each project team consists of 9 employees as described. On how many of the 49 project teams do you expect there to be exactly 3 computer science graduates? Give your answer to 1 decimal place. ___

Part e) I meet 30 employees at random. What is the probability that the second employee I meet is the first one who is a computer science graduate? Give your answer to 3 decimal places. ___

Part f) I meet 85 employees at random on a daily basis. What is the mean number of computer science graduates among the 85 that I meet? Give your answer to one decimal place. ___

Solution: Part a) We have $X \sim \text{Binomial}(9, 0.75)$. Hence $P(X = 9) = p^9 = 0.75^9 = 0.0751$.

Part b) $P(X = 3) = \binom{9}{3} \cdot p^3 \cdot (1-p)^{(9-3)} = \binom{9}{3} \cdot 0.75^3 \cdot (0.25)^6 = 0.0087$.

Part c) The trick is to find a relationship between $P(X = x)$ and $P(X = x - 1)$.

$$\begin{aligned}
\frac{P(X=x)}{P(X=x-1)} &= \frac{\binom{9}{x} p^x (1-p)^{9-x}}{\binom{9}{x-1} p^{x-1} (1-p)^{9-(x-1)}} \\
&= \frac{\frac{9!}{(9-x)!x!} p^x (1-p)^{9-x}}{\frac{9!}{(9-(x-1))!(x-1)!} p^{x-1} (1-p)^{9-(x-1)}} \\
&= \frac{\frac{9!}{(9-x)!x(x-1)!} p^x (1-p)^{9-x}}{\frac{9!}{(9-(x-1))!(x-1)!} p^{x-1} (1-p)(1-p)^{9-x)}} \\
&= \frac{x-9+1}{x} \cdot \frac{p}{1-p}
\end{aligned}$$

We want to solve for x such that $\frac{P(X=x)}{P(X=x-1)} > 1$. That is:

$$\begin{aligned}
\frac{P(X=x)}{P(X=x-1)} &> 1 \\
\frac{x-9+1}{x} \cdot \frac{p}{1-p} &> 1
\end{aligned}$$

So

$$\begin{aligned}
p \cdot (9+1-x) &> x(1-p) \\
p \cdot (9+1) &> x \\
0.75 \cdot 10 &> x
\end{aligned}$$

So $x < 7.5$. Since x must be an integer, we have $x = 7$.

Part d) Let W be the number of project teams with exactly 3 computer science graduates.

$$W \sim \text{Binomial}(49, p = 0.0087)$$

Then

$$\begin{aligned} E(W) &= n \cdot p \\ &= 49 \cdot 0.0087 \\ &= 0.43 \end{aligned}$$

Part e) Let S be a random variable counting the number of people I meet until I meet the first computer science graduate.

$$S \sim \text{Geometric}(p = 0.75)$$

So

$$\begin{aligned} P(X = 2) &= (1 - p)^{2-1} \cdot p \\ &= (1 - 0.75)^1 \cdot 0.75 \\ &= 0.1875 \end{aligned}$$

Part f) Let T be a random variable counting the number of computer science graduates I meet daily.

$$T \sim \text{Binomial}(85, p = 0.75)$$

So

$$\begin{aligned} E(T) &= n \cdot p \\ &= 85 \cdot 0.75 \\ &= 63.75 \end{aligned}$$

Correct Answers:

- 0.0751
- 0.0087
- 7
- 0.43
- 0.1875
- 63.75

+6pc-1pc

Problem 15

In the game of roulette, a steel ball is rolled onto a wheel that contains 18 red, 18 black, and 2 green slots. If the ball is rolled 18 times, find the probability of the following events.

A. The ball falls into the green slots 5 or more times.

Probability = ______

B. The ball does not fall into any green slots.

Probability = ______

C. The ball falls into black slots 13 or more times.

Probability = ______

D. The ball falls into red slots 11 or fewer times.

Probability = ______

Correct Answers:

- 0.00194087567881314
- 0.377868138992545
- 0.0295542990583112
- 0.920093685834871

+6pc-1pc
Problem 16

In the United States, voters who are neither Democrat nor Republican are called Independent. It is believed that 9% of voters are Independent. A survey asked 31 people to identify themselves as Democrat, Republican, or Independent.

A. What is the probability that none of the people are Independent?

Probability = ______

B. What is the probability that fewer than 6 are Independent?

Probability = ______

C. What is the probability that more than 2 people are Independent?

Probability = ______

Correct Answers:

- 0.0537382057688453
- 0.94476981511102
- 0.537083092800317

+6pc-1pc
Problem 17

A sign on the pumps at a gas station encourages customers to have their oil checked, and claims that one out of 5 cars needs to have oil added. If this is true, what is the probability of each of the following:

A. One out of the next four cars needs oil.

Probability = ______

B. Two out of the next eight cars needs oil.

Probability = ______

C. 10 out of the next 40 cars needs oil.

Probability = ______

Correct Answers:

- 0.4096
- 0.29360128
- 0.107453737710898

+6pc-1pc
Problem 18

In the game of blackjack as played in casinos in Las Vegas, Atlantic City, Niagara Falls, as well as many other cities, the dealer has the advantage. Most players do not play very well. As a result, the probability that the average player wins a hand is about 0.3. Find the probability that an average player wins

A. twice in 5 hands.

Probability = ______

B. 9 or more times in 24 hands.

Probability = ______

There are several books that teach blackjack players the ”basic strategy” which increases the probability of winning any hand to 0.43. Assuming that the player plays the basic strategy, find the probability that he or she wins

C. twice in 5 hands.

Probability = ______

D. 9 or more times in 24 hands.

Probability = ______

Correct Answers:

- 0.3087
- 0.274962968395945
- 0.342421857
- 0.771782995255851

+6pc-1pc

Problem 19

A couple decided to have 4 children.

(a) What is the probability that they will have at least two boys? ___

(b) What is the probability that all the children will be of the same sex? ___

Correct Answers:

- 0.6875
- 0.125

+6pc-1pc

Problem 20

If x is a binomial random variable, compute the mean, the standard deviation, and the variance for each of the following cases:

(a) $n = 5, p = 0.2$
$\mu =$ _____
$\sigma^2 =$ _____
$\sigma =$ _____

(b) $n = 5, p = 0.3$
$\mu =$ _____
$\sigma^2 =$ _____
$\sigma =$ _____

(c) $n = 6, p = 0.4$
$\mu =$ _____
$\sigma^2 =$ _____
$\sigma =$ _____

(d) $n = 6, p = 0.4$
$\mu =$ _____
$\sigma^2 =$ _____
$\sigma =$ _____

Correct Answers:

- 1
- 0.8
- 0.894427190999916
- 1.5
- 1.05
- 1.02469507659596
- 2.4
- 1.44
- 1.2
- 2.4
- 1.44
- 1.2

+6pc-1pc
Problem 21

A quiz consists of 20 multiple-choice questions, each with 4 possible answers. For someone who makes random guesses for all of the answers, find the probability of passing if the minimum passing grade is 50 %.
$P(\text{pass}) =$ ______

Correct Answers:

- 0.0138644

+6pc-1pc
Problem 22

If x is a binomial random variable, compute $P(x)$ for each of the following cases:

(a) $P(x \leq 2), n = 9, p = 0.7$

$P(x) =$ ____

(b) $P(x > 5), n = 9, p = 0.2$

$P(x) =$ ____

(c) $P(x < 3), n = 4, p = 0.7$

$P(x) =$ ____

(d) $P(x \geq 2), n = 4, p = 0.7$

$P(x) =$ ____
Correct Answers:

- 0.00429089399999999
- 0.003066368
- 0.3483
- 0.9163

+6pc-1pc
Problem 23

In each part, assume the random variable X has a binomial distribution with the given parameters. Compute the probability of the event.

(a) $n = 5, p = 0.2$
$Pr(X = 5) =$ ____

(b) $n = 3, p = 0.3$
$Pr(X = 3) =$ ____

(c) $n = 6, p = 0.5$
$Pr(X = 0) =$ ____

(d) $n = 4, p = 0.1$
$Pr(X = 2) =$ ____
Correct Answers:

- 0.00032
- 0.027
- 0.015625
- 0.0486

+6pc-1pc
Problem 24

The rates of on-time flights for commercial jets are continuously tracked by the U.S. Department of Transportation. Recently, Southwest Air had the best rate with 80 % of its flights arriving on time. A test is conducted by randomly selecting 16 Southwest flights and observing whether they arrive on time.

(a) Find the probability that at least 13 flights arrive late.

____

(b) Would it be unusual for Southwest to have at least 9 flights arrive on time? [?/yes/no]
(Here "unusual" means "probability < 0.05").

Note: In part (a) enter **C(n,k)** to stand for the binomial coefficient $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. For example $C(6, 2) = 15$ is the number of ways of choosing 2 things out of a set of 6 things, if order doesn't count. Don't enter the factorial formula $\dfrac{n!}{k!(n-k)!}$ if n is large because large factorials are huge and will choke the computer!
Correct Answers:

- $2.47883 \times 10^{-7} + \mathrm{C}(16, 16) \cdot 0.2^{16} \cdot 0.8^{16-16}$
- yes

+6pc-1pc
Problem 25

A man claims to have extrasensory perception (ESP). As a test, a fair coin is flipped 23 times, and the man is asked to predict the outcome in advance. He gets 17 out of 23 correct. What is the probability that he would have done at least this well if he had no ESP?

Probability = ________
Solution:

SOLUTION

Let X be the random variable that gives the number of correct guesses the ma n would make without ESP. Then X is a binomial random variable with paramet ers $n = 23$ and $p = 0.5$ (since that is the probability of successfully g uessing whether a fair coin will land on heads or tails. Hence the probability he would do at least as well without ESP is

$$P(X \geq 17) = \sum_{i=17}^{23} \binom{23}{i} (0.5)^i (0.5)^{n-i} = (0.5)^{23} \sum_{i=17}^{23} \binom{23}{i} \approx 0.01734483.$$

Correct Answers:

- 0.0173448324203491

+6pc-1pc
Problem 26

The Census Bureau reports that 82% of Americans over the age of 25 are high school graduates. A survey of randomly selected residents of certain county included 1380 who were over the age of 25, and 1138 of them were high school graduates.

(a) Find the mean and standard deviation for the number of high school graduates in groups of 1380 Americans over the age of 25.

Mean = ______

Standard deviation = ______

(b) Is that county result of 1138 unusually high, or low, or neither?

(Enter HIGH or LOW or NEITHER) ______
Correct Answers:

- 1131.6
- 14.2719304931043
- neither

+6pc-1pc
Problem 27

Consider $n = 7$ independent flips of a fair coin. Say that a changeover occurs whenever an outcome differs from the one preceding it. For example, if $n = 6$ and the outcome is $T\ H\ T\ T\ H\ T$, then there is a total of 4 changeovers. Find the expected number of changeovers for $n = 7$.

Answer = ________
Correct Answers:

- 3