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Problem 1

Two dice are tossed 468 times. How many times would you expect to get a sum of 4?

Answer: _____
Correct Answers:

- $468 \cdot \frac{3}{36}$

+6pc-1pc
Problem 2

A box contains 5 red and 7 green marbles. If 9 marbles are drawn without replacement, what is the expected number of red marbles?

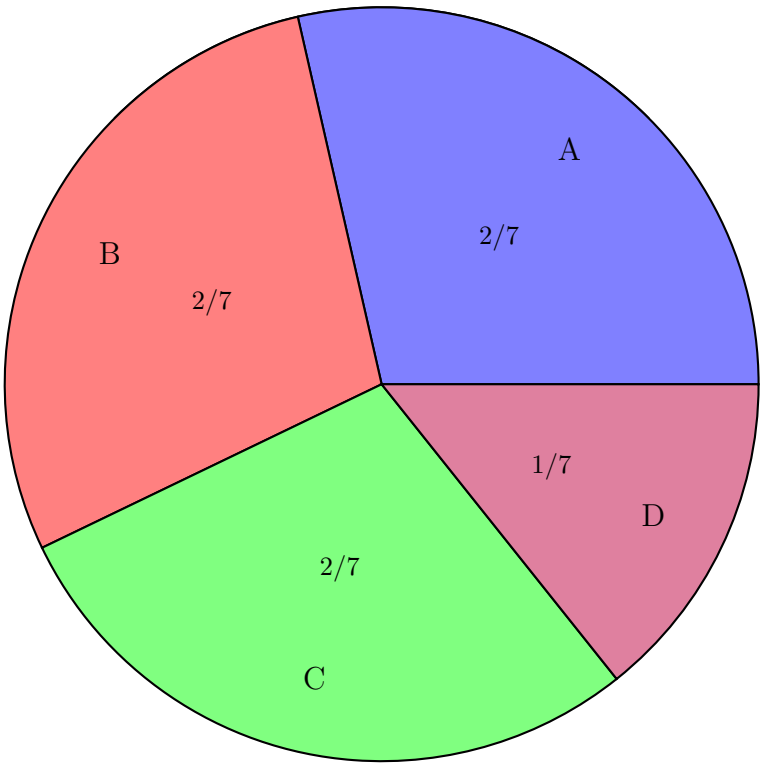
Answer: _____
Correct Answers:

- 3.75

+6pc-1pc
Problem 3

Consider the following game of chance based on the spinner below:

Each spin costs \$2. If the spinner lands on C the player wins \$8. If the spinner stops on A the player wins a quarter. Otherwise, the player wins nothing.



Calculate the player's expected profit. __

Note: Express your answer to at least three decimal places in dollar form.

Correct Answers:

- 0.357143

+6pc-1pc

Problem 4

A charity holds a raffle in which each ticket is sold for $20. A total of 11000 tickets are sold. They raffle one grand prize which is a Mercedes Benz E350 valued at $50000 along with 6 second prizes of Honda motorcycles valued at $11000 each. What are the expected winnings for a single ticket buyer? **Express to at least three decimal place accuracy in dollar form (as opposed to cents).**

Answer: $ _____

Correct Answers:

- −9.45454545454546

+6pc-1pc

Problem 5

Find the expected value for the random variable:

X	$P(X)$
3	0.19
5	0.24
6	0.12
7	0.45

$E(X) =$ _____

Correct Answers:

- 5.64

+6pc-1pc

Problem 6

Suppose that from a standard deck, you draw three cards without replacement. What is the expected number of hearts that you will draw?

_____

Correct Answers:

- 0.75

+6pc-1pc

Problem 7

A fair die is rolled once, and the number score is noted. Let the random variable X be twice this score. Define the variable Y to be zero if an odd number appears and X otherwise. By finding the probability mass function in each case, find the expectation of the following random variables:

Please answer to 3 decimal places.

Part a) X

—

Part b) Y

—

Part c) $X + Y$

—

Part d) XY

—

Solution:

Part a)

The probability mass function here is

x	2	4	6	8	10	12
P(X=x)	1/6	1/6	1/6	1/6	1/6	1/6

Hence $E(X) = \frac{1}{6}(2 + 4 + 6 + 8 + 10 + 12) = 7$.

Part b)

The probability mass function here is

y	0	4	8	12
P(Y=y)	1/2	1/6	1/6	1/6

Hence $E(Y) = 0 + \frac{1}{6}(4 + 8 + 12) = 4$.

Part c)

Setting Z = X+Y we have

z	2	6	8	10	16	24
P(Z=z)	1/6	1/6	1/6	1/6	1/6	1/6

and so
$E(Z) = \frac{1}{6}(2 + 6 + 8 + 10 + 16 + 24) = 11.$

Part d)

Letting W = XY we have

w	0	16	64	144
P(W=w)	1/2	1/6	1/6	1/6

and so
$E(W) = \frac{1}{6}(16 + 64 + 144) = \frac{112}{3}$.

Correct Answers:

- 7
- 4
- 11
- 37.3333

+6pc-1pc
Problem 8

Two balls are selected at random without replacement from a set that contains five balls labelled 1, 1, 2, 2, and 3 respectively. Let X denote the sum and Y the maximum of the two numbers drawn. By finding the probability mass function in each case, find the expectation of the following random variables:

Please answer to 3 decimal places.

Part a) X

—

Part b) Y

—

Part c) $X + Y$

—

Part d) XY

—

Solution:

Although there are $\binom{5}{2} = 10$ ways to select the two balls, there are only five possible different pairings. There is one way to obtain each of (1,1) and (2,2), four ways to get (1,2), two ways for (1,3), and two for (2,3).
Part a)

The probability mass function here is

x	2	3	4	5
P(X=x)	1/10	4/10	3/10	2/10

Hence $E(X) = 2 \times \frac{1}{10} + 3 \times \frac{2}{5} + 4 \times \frac{3}{10} + 5 \times \frac{1}{5} = \frac{18}{5}$.

Part b)

The probability mass function here is

y	1	2	3
P(Y=y)	1/10	5/10	4/10

Hence $E(Y) = \frac{1}{10} + 2 \times \frac{1}{2} + 3 \times \frac{2}{5} = \frac{23}{10}$.

Part c)

Setting Z = X+Y we have

z	3	5	6	7	8
P(Z=z)	1/10	4/10	1/10	2/10	2/10

and so $E(Z) = 3 \times \frac{1}{10} + 5 \times \frac{2}{5} + 6 \times \frac{1}{10} + 7 \times \frac{1}{5} + 8 \times \frac{1}{5} = \frac{59}{10}$.

Part d)

Letting W = XY we have

w	2	6	8	12	15
P(W=w)	1/10	4/10	1/10	2/10	2/10

and so $E(W) = 2 \times \frac{1}{10} + 6 \times \frac{2}{5} + 8 \times \frac{1}{10} + 12 \times \frac{1}{5} + 15 \times \frac{1}{5} = \frac{44}{5}$.

Correct Answers:

- 3.6
- 2.3
- 5.9
- 8.8

+6pc-1pc

Problem 9

The discrete random variable X has a probability distribution defined below:

x	0	2	4	8
P(X=x)	1/4	1/8	1/2	1/8

The expectation of X is:

- A. 13/4
- B. 2/3
- C. 7/2
- D. 3
- E. 14

Solution:

The expectation is

Correct Answers:

- A

+6pc-1pc

Problem 10

The owner of a small firm has just purchased a personal computer, which she expects will serve her for the next two years. The owner has been told that she "must" buy a surge suppressor to provide protection for her new hardware against possible surges or variations in the electrical current, which have the capacity to damage the computer. The amount of damage to the computer depends on the strength of the surge. It has been estimated that there is a 1% chance of incurring 400 dollar damage, 5% chance of incurring 300 dollar damage, and 11% chance of incurring 175 dollar damage from a surge within the next two years. An inexpensive suppressor, which would provide protection for only one surge, can be purchased. How much should the owner be willing to pay if she makes decisions on the basis of expected value?

Expected value = _____

Solution:

SOLUTION

Let the random variable D be the amount of damage to the computer (in dollars) caused by a surge within the next two years.
Then

$$\begin{aligned} E[D] &= 400 \cdot P(D = 400) + 300 \cdot P(D = 300) + 175 \cdot P(D = 175) \\ &= 400 \cdot 0.01 + 300 \cdot 0.05 + 175 \cdot 0.11 \\ &= 38.25 \end{aligned}$$

Hence the owner expects her computer to incur $38.25 of damage from a surge in the next two years and so should be willing to pay $38.25 for a surge protector.

Correct Answers:

- 38.25

+6pc-1pc
Problem 11

A rock concert producer has scheduled an outdoor concert. The producer estimates the attendance will depend on the weather according to the following table.

Weather	Attendance	Probability
wet, cold	6000	0.2
wet, warm	30000	0.1
dry, cold	15000	0.1
dry, warm	45000	0.6

(a) What is the expected attendance?

answer: ____

(b) If tickets cost $ 25 each, the band will cost $ 300000, plus $ 50000 for administration. What is the expected profit?

answer: ____
Correct Answers:

- 32700

- 467500

+6pc-1pc
Problem 12

Four buses carrying 155 high school students arrive to Montreal. The buses carry, respectively, 37, 46, 29, and 43 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus. Compute the expectations of X and Y:

$E(X) =$_____

$E(Y) =$_____
Correct Answers:

- 39.8387096774194
- 38.75

+6pc-1pc
Problem 13

A fair die is rolled 11 times. What is the expected sum of the 11 rolls?

____
Correct Answers:

- 38.5

+6pc-1pc
Problem 14

For a random variable X, suppose that $E[X] = 3$ and $\mathrm{Var}(X) = 7$. Then

(a) $E[(4 + X)^2] =$ _____

(b) $\mathrm{Var}(5 + 5X) =$ _____
Solution: **(a)** We have

$$\begin{aligned} E[(4 + X)^2] = E[16 + 8X + X^2] &= 16 + 8(3) + E[X^2] \\ &= 40 + (\mathrm{Var}(X) + E[X]^2) \\ &= 40 + (7 + 9) = 56. \end{aligned}$$

(b) Here,

$$\mathrm{Var}(5 + 5X) = 25\,\mathrm{Var}(X) = 175.$$

Correct Answers:

- $16 + 24 + 7 + 9$
- $25 \cdot 7$

+6pc-1pc

Problem 15

Suppose that a pair of dice is tossed. One die has 8 sides, the other has 6 sides. What is the expected value of the sum of the two dice?

Answer = _____
Correct Answers:

- 8

+6pc-1pc
Problem 16

When parking a car in a downtown parking lot, drivers pay according to the number of hours or fraction thereof. The probability distribution of the number of hours cars are parked has been estimated as follows:

X	1	2	3	4	5	6	7	8
P(X)	0.206	0.144	0.118	0.09	0.07	0.035	0.04	0.297

A. Mean = _____

B. Standard Deviation = _____

The cost of parking is 2.5 dollars per hour. Calculate the mean and standard deviation of the amount of revenue each car generates.

A. Mean = _____

B. Standard Deviation = _____

Correct Answers:

- 4.424
- 2.77312531271127
- 11.06
- 6.93281328177819

+6pc-1pc
Problem 17

There are four activities along the critical path for a project. The expected values and variances of the completion times of the activities are listed below. Determine the expected value and variance of the completion time of the project.

Activity	Expected Completion Time (Days)	Variance
1	17	6
2	12	3
3	25	5
4	6	1

Expected value of completion time of project = _____

Variance of completion time of project = _____
Correct Answers:

- 60
- 15

+6pc-1pc
Problem 18

Find the mean, variance and standard deviation for the probability distribution given below:

X	-1	7	8	9
P(X)	0.575	0.108	0.203	0.114

A. Mean = _____

B. Variance = _____

C. Standard Deviation = _____

Correct Answers:

- 2.831
- 20.078439
- 4.48089711999729

+6pc-1pc
Problem 19

The mean and standard deviation of a random variable x are 11 and 3 respectively. Find the mean and standard deviation of the given random variables:

(1) $y = x + 5$
$\mu =$ ___
$\sigma =$ ___

(2) $v = 2x$
$\mu =$ ___
$\sigma =$ ___

(3) $w = 2x + 5$
$\mu =$ ___
$\sigma =$ ___
Correct Answers:

- 16
- 3
- 22
- 6

- 27
- 6

+6pc-1pc
Problem 20

Four buses carrying 156 high school students arrive to Montreal. The buses carry, respectively, 32, 48, 31, and 45 students. One of the studetns is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus. Compute the expectations and variances of X and Y:

$E(X) =$_____

$\text{Var}(X) =$_____

$E(Y) =$_____

$\text{Var}(Y) =$_____
Correct Answers:

- 40.474358974359
- 55.9031886916502
- 39
- 57.5

+6pc-1pc
Problem 21

Prizes and the chances of winning in a sweepstakes are given in the table below.

Prize	Chances
\$15,000,000	1 chance in 200,000,000
\$250,000	1 chance in 100,000,000
\$50,000	1 chance in 70,000,000
\$10,000	1 chance in 4,000,000
\$300	1 chance in 200,000
A watch valued at \$95	1 chance in 3,000

(a) Find the expected value (in dollars) of the amount won by one entry.

_____

(b) Find the expected value (in dollars) if the cost of entering this sweepstakes is the cost of a postage stamp (34 cents)

_____

Solution:

SOLUTION

(a) Let W be the amount of money (in dollars) won by an entry. Then

$$\begin{aligned} E[W] =& 15000000 \cdot P(W = 15000000) + 250000 \cdot P(W = 250000) + 50000 \cdot P(W = 50000) \\ &+ 10000 \cdot P(W = 10000) + 300 \cdot P(W = 300) + 95 \cdot P(W = 95) \\ &= \frac{15000000}{200000000} + \frac{250000}{100000000} + \frac{50000}{70000000} + \frac{10000}{4000000} + \frac{300}{200000} + \frac{95}{3000} \\ &= 0.113880952380952 \end{aligned}$$

So the expected value of the amount won by one entry in the sweepstakes is about 11 cents.

(b) Let X be the net earnings from one entry into the sweepstakes. Then $X = W - 0.34$, so

$$E[X] = E[W - 0.34] = E[W] - 0.34 = -0.226119047619048.$$

Hence, taking into account the cost of postage, there is an expected loss of about 23 cents from one entry into the sweepstakes.

Correct Answers:

- 0.113880952380952
- −0.226119047619048

+6pc-1pc
Problem 22

If $E[X] = -3$ and $\mathrm{Var}(X) = 5$, then
$E[(2 + 5X)^2] =$_____
and
$\mathrm{Var}(4 + 5X) =$_____.
Correct Answers:

- 294
- 125