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Problem 1

Which of the following is/are NOT correct about a continuous random variable? CHECK ALL THAT APPLY.

- A. The probability that it has a value that falls between real values a and b is given by the area under the density function over the range (a, b).
- B. It can only take on integer values.
- C. The probability that it has a value equal to x is given by $f(x)$ where f is the density function.

Correct Answers:

- BC

+6pc-1pc
Problem 2

The amount of time it takes for a student to complete a statistics quiz is uniformly distributed (or, given by a random variable that is uniformly distributed) between 25 and 64 minutes. One student is selected at random. Find the probability of the following events.

A. The student requires more than 59 minutes to complete the quiz.
Probability = ______

B. The student completes the quiz in a time between 29 and 35 minutes.
Probability = ______

C. The student completes the quiz in exactly 44.35 minutes.
Probability = ______

Correct Answers:

- $(64-59)\,\frac{1}{64-25}$
- $(35-29)\,\frac{1}{64-25}$
- 0

+6pc-1pc
Problem 3

Suppose that random variable X is uniformly distributed between 7 and 25. Draw a graph of the density function, and then use it to help find the following probabilities:

A. $P(X > 25) =$ ______

B. $P(X < 13.5) =$ ______

C. $P(10 < X < 21) =$ ______

D. $P(17 < X < 28) =$ ______

Correct Answers:

- 0
- 0.361111111111111
- 0.611111111111111
- 0.444444444444444

+6pc-1pc
Problem 4

Suppose x is a random variable best described by a uniform probability that ranges from 0 to 4. Compute the following:

(a) the probability density function $f(x) =$ ____

(b) the mean $\mu =$ ____

(c) the standard deviation $\sigma =$ ____

(d) $P(\mu - \sigma \leq x \leq \mu + \sigma) =$ ____

(e) $P(x \geq 0.92) =$ ____
Correct Answers:

- 0.25
- 2
- 1.15470053837925
- 0.577350269189626
- 0.77

+6pc-1pc
Problem 5

Suppose the time to process a loan application follows a uniform distribution over the range 5 to 16 days. What is the probability that a randomly selected loan application takes longer than 11 days to process?

answer: ____
Correct Answers:

- 0.454545454545455

+6pc-1pc

Problem 6

If a is uniformly distributed over $[-21, 31]$, what is the probability that the roots of the equation

$$x^2 + ax + a + 63 = 0$$

are both real? ____
Correct Answers:

- 0.384615384615385

+6pc-1pc
Problem 7

A manager of an apartment store reports that the time of a customer on the second floor must wait for the elevator has a uniform distribution ranging from 1 to 4 minutes. If it takes the elevator 15 seconds to go from floor to floor, find the probability that a hurried customer can reach the first floor in less than 3.5 minutes after pushing the elevator button on the second floor.

answer : ____
Correct Answers:

- 0.75

+6pc-1pc
Problem 8

Suppose a random variable x is best described by a uniform probability distribution with range 0 to 4. Assume $0 \le a \le 4$. For each of the following probability statements, find the value of a that makes the statement true.

(a) $P(x \le a) = 0.87$
$a =$ ___

(b) $P(x < a) = 0.08$
$a =$ ___

(c) $P(x \ge a) = 0.56$
$a =$ ___

(d) $P(x > a) = 0.57$
$a =$ ___

(e) $P(1.58 \le x \le a) = 0.45$
$a =$ ___

Solution: If $0 \le c \le d \le 4$ the probability that x is between c and d is $\dfrac{d-c}{4-0} = \dfrac{d-c}{4}$. So

(a) $\frac{a-0}{4} = 0.87$ so $a = 3.48$

(b) $\frac{a-0}{4} = 0.08$ so $a = 0.32$

(c) $\frac{4-a}{4} = 0.56$ so $a = 1.76$

(d) $\frac{4-a}{4} = 0.57$ so $a = 1.72$

(e) $\frac{a-1.58}{4} = 0.45$ so $a = 3.38$

Correct Answers:

- 3.48
- 0.32
- 1.76
- 1.72
- 3.38

+6pc-1pc

Problem 9

The weather in Rochester in December is fairly constant. Records indicate that the low temperature for each day of the month tend to have a uniform distribution over the interval 15° to 35°F. A business man arrives on a randomly selected day in December.

(a) What is the probability that the temperature will be above 27°?
answer: ____

(b) What is the probability that the temperature will be between 17° and 28°?
answer: ____

(c) What is the expected temperature?
answer: ____

Correct Answers:

- 0.4
- 0.55
- 25

+6pc-1pc

Problem 10

Two points along a straight stick of length 37 cm are randomly selected. The stick is then broken at those two points. Find the probability that all of the resulting pieces have lenght at least 5.5 cm.

probability = ________

Correct Answers:

- 0.306975894813733

+6pc-1pc

Problem 11

A man and a woman agree to meet at a cafe about noon. If the man arrives at a time uniformly distributed between 11 : 40 and 12 : 10 and if the woman independently arrives at a time uniformly distributed between 11 : 55 and 12 : 55, what is the probability that the first to arrive waits no longer than 15 minutes?

____

Correct Answers:

- 0.25

+6pc-1pc

Problem 12

Two points are selected randomly on a line of length 40 so as to be on opposite sides of the midpoint of the line. In other words, the two points X and Y are independent random variables such that X is uniformly distributed over $[0, 20)$ and Y is uniformly distributed over $(20, 40]$. Find the probability that the distance between the two points is greater than 3.
answer: ________
Correct Answers:

- 0.98875

+6pc-1pc
Problem 13

x and y are uniformly distributed over the interval $[0, 1]$. Find the probability that $|x - y|$, the distance between x and y, is less than 0.25.

____
Correct Answers:

- 0.4375

+6pc-1pc
Problem 14

Which of the following are true about all normal distributions? *Check all that apply*

- A. They are defined by the mean and standard deviation.
- B. They have one peak.
- C. They have no major outliers.
- D. They are symmetric.

The z-score corresponding to an observed value of a variable tells you the number of standard deviations that the observation is from the mean

- A. True
- B. False

A positive z-score indicates that the observation is

- A. above the mean
- B. below the mean

Correct Answers:

- ABCD

- A
- A

+6pc-1pc
Problem 15

Consider a normal distribution curve where 90-th percentile is at 20 and the 45-th percentile is at 5. Use this information to find the mean, μ , and the standard deviation, σ , of the distribution.

a) $\mu =$ ________

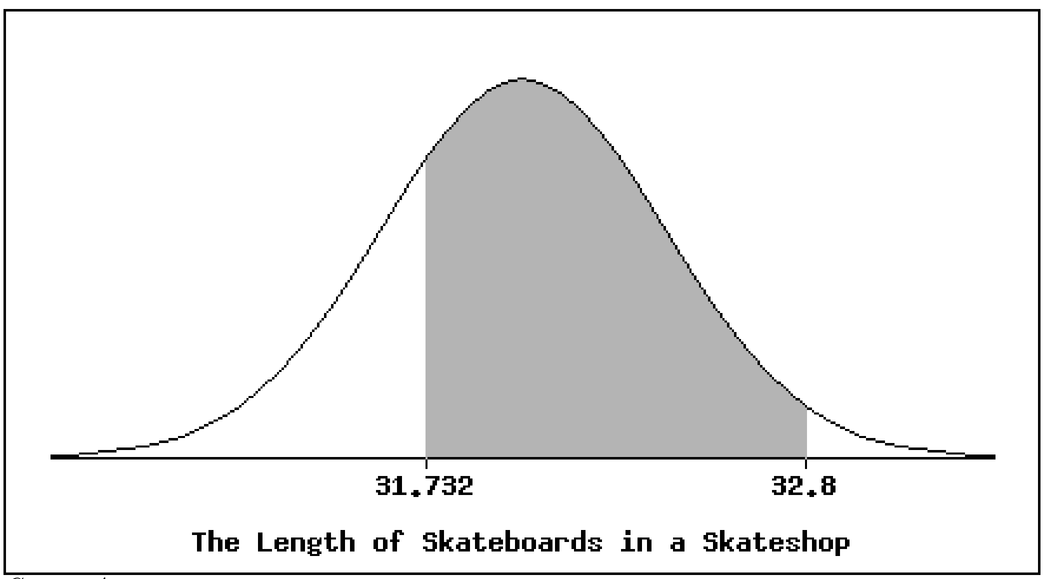
b) $\sigma =$ ________

Correct Answers:

- 6.33946863689951
- 10.6593822816905

+6pc-1pc
Problem 16

Length of skateboards in a skateshop are normally distributed with a mean of 32 in and a standard deviation of 0.4 in. The figure below shows the distribution of the length of skateboards in a skateshop. Calculate the shaded area under the curve. **Express your answer in decimal form with at least two decimal place accuracy.**
Answer: ______



Correct Answers:

- 0.7258208

+6pc-1pc

Problem 17

Find the normalizing constant c so that $\int_{-\infty}^{\infty} ce^{-x^2/2}\,dx = 1$.
$c =$ ________

Correct Answers:

- 0.398942280401433

+6pc-1pc
Problem 18

Suppose that X is normally distributed with mean 105 and standard deviation 17.

A. What is the probability that X is greater than 132.71?
Probability = ______

B. What value of X does only the top 10% exceed?
$X =$ ______
Correct Answers:

- 0.0515507
- 126.78635

+6pc-1pc
Problem 19

Select True or False from each pull-down menu, depending on whether the corresponding statement is true or false.

$\boxed{?}$ 1. A random variable X is normally distributed with a mean of 150 and a variance of 36. Given that $X = 120$, its corresponding $z-$ score is 5.0

$\boxed{?}$ 2. Let z_1 be a $z-$score that is unknown but identifiable by position and area. If the symmetrical area between $-z_1$ and $+z_1$ is 0.9544, the value of z_1 is 2.0

$\boxed{?}$ 3. The mean and standard deviation of an exponential random variable are equal to each other.

$\boxed{?}$ 4. Let z_1 be a $z-$ score that is unknown but identifiable by position and area. If the area to the right of z_1 is 0.8413, the value of z_1 is 1.0

Correct Answers:

- F
- T
- T
- F

+6pc-1pc

Problem 20

Consider two normal distributions, one with mean -14 and standard deviation 10, the other with mean 3 and standard deviation 10. Answer the following statements using **true** or **false**.

a) The two distributions have the same shape.
answer: ________

b) The two distributions are centered at the same place.
answer: ________

Correct Answers:

- true
- false

+6pc-1pc

Problem 21

Find the following probabilities for the standard normal random variable z:

(a) $P(-1.32 \leq z \leq 0.58) =$ ____

(b) $P(-1.37 \leq z \leq 2.14) =$ ____

(c) $P(z \leq 1.94) =$ ____

(d) $P(z > -1.98) =$ ____

Correct Answers:

- 0.625626
- 0.89848
- 0.97381
- 0.976148

+6pc-1pc

Problem 22

Which of the following normal distributions has the widest spread?

- A. A normal distribution with mean 3 and standard deviation 2
- B. A normal distribution with mean 1 and standard deviation 3
- C. A normal distribution with mean 2 and standard deviation 1
- D. A normal distribution with mean 0 and standard deviation 2
- E. None of the above

Correct Answers:

- B

+6pc-1pc

Problem 23

What are the parameters for a normal curve?

- A. the population median and population standard deviation
- B. the sample mean and sample standard deviation
- C. the population mean and population standard deviation
- D. the population mean and population variance
- E. None of the above

Correct Answers:

- C

+6pc-1pc

Problem 24

The shelf life of a battery produced by one major company is known to be Normally distributed, with a mean life of 3 years and a standard deviation of 0.1 years.

What value of shelf life do 16% of the battery shelf lives fall below? Round your answer to one decimal place.

Answer: _______ years.

Correct Answers:

- 2.9

+6pc-1pc

Problem 25

The lengths of a certain type of chain are approximately Normally distributed with a mean of 3.2 cm and a standard deviation of 0.4 cm.

Find the value of ℓ such that $P(L > \ell) = 0.01$

- A. 3.37 cm
- B. 4.13 cm
- C. 0.40 cm
- D. 7.86 cm
- E. 3.20 cm

Solution: We want a length ℓ such that $P(L < \ell) = 0.99$. Transforming this to a standard normal distribution, this gives $P(Z < \frac{\ell-3.2}{0.4}) = 0.99$. Looking up the inverse probability for the standard normal distribution, we have that $\frac{\ell-3.2}{0.4} = 2.33$. Thus $\ell = 4.13$ cm (choice B).

Correct Answers:

- B

+6pc-1pc

Problem 26

Suppose a car manufacturer believes its windscreen wipers will last on average for three years on their cars if driven by a typical driver in the province. Moreover, the manufacturer believes the lifetime of the wipers under such conditions is Normally distributed with a standard deviation of two years. Find the probability that, if on a car driven by a typical driver, a windscreen wiper lasts for a time that is not within 1.3 years of the mean lifetime.

The probability is: __

Solution:

If X denotes the time the windscreen wiper lasts, then $X \sim N(3,4)$ and we wish to find $P(|X-3| > 1.3)$. If $Z \sim N(0,1)$, we have

$P(|X-3| > 1.3)$
$= P(X > 4.3) + P(X < 1.7)$
$= P(Z > \frac{4.3-3}{2}) + P(Z < \frac{1.7-3}{2})$
$= P(Z > 0.65) + P(Z < -0.65)$
$= (1 - P(Z < 0.65)) + P(Z < -0.65)$
$= 0.2578461 + 0.2578461$
$= 0.52$
to 2 decimal places.;

Correct Answers:

- 0.5156922

+6pc-1pc
Problem 27

Scores on a certain intelligence test for children between ages 13 and 15 years are approximately normally distributed with $\mu = 106$ and $\sigma = 15$.

(a) What proportion of children aged 13 to 15 years old have scores on this test above 88 ? (NOTE: Please enter your answer in decimal form. For example, 45.23% should be entered as 0.4523.)
Answer: ____

(b) Enter the score which marks the lowest 25 percent of the distribution.
Answer: ____

(c) Enter the score which marks the highest 5 percent of the distribution.
Answer: ____
Correct Answers:

- 0.88493
- 95.8826
- 130.673

+6pc-1pc
Problem 28

The number of pizzas consumed per month by university students is normally distributed with a mean of 14 and a standard deviation of 3.

A. What proportion of students consume more than 15 pizzas per month?

Probability = ______

B. What is the probability that in a random sample of size 10, a total of more than 130 pizzas are consumed? (Hint: What is the mean number of pizzas consumed by the sample of 10 students?)

Probability = ______

Correct Answers:

- 0.369441
- 0.85408

+6pc-1pc
Problem 29

An automatic machine in a manufacturing process is operating properly if the lengths of an important subcomponent are normally distributed with a mean of 119 cm and a standard deviation of 4.9 cm.

A. Find the probability that one selected subcomponent is longer than 121 cm.

Probability = ______

B. Find the probability that if 3 subcomponents are randomly selected, their mean length exceeds 121 cm.

Probability = ______

C. Find the probability that if 3 are randomly selected, all 3 have lengths that exceed 121 cm.

Probability = ______

Correct Answers:

- 0.341577
- 0.239796
- 0.039853444189467

+6pc-1pc
Problem 30

Assume that women's weights are normally distributed with a mean given by $\mu = 143$ lb and a standard deviation given by $\sigma = 29$ lb.

(a) If 1 woman is randomly selected, find the probabity that her weight is between 113 lb and 175 lb

____

(b) If 6 women are randomly selected, find the probability that they have a mean weight between 113 lb and 175 lb

____

(c) If 84 women are randomly selected, find the probability that they have a mean weight between 113 lb and 175 lb

____

Correct Answers:

- 0.714629
- 0.99092398
- 1

+6pc-1pc
Problem 31

Healty people have body temperatures that are normally distributed with a mean of $98.20°F$ and a standard deviation of $0.62°F$.

(a) If a healthy person is randomly selected, what is the probability that he or she has a temperature above $98.7°F$?

answer: ____

(b) A hospital wants to select a minimum temperature for requiring further medical tests. What should that temperature be, if we want only 1 % of healty people to exceed it?

answer: ____
Correct Answers:

- 0.209991
- 99.642337

+6pc-1pc
Problem 32

Sam's bowling scores are approximately normally distributed with mean 100 and standard deviation 25, while Leo's scores are normally distributed with mean 165 and standard deviation 14. If Sam and Leo each bowl one game, then assuming that their scores are independent random variables, approximate the probability that the total of their scores is above 250.

____
Correct Answers:

- 0.699688

+6pc-1pc
Problem 33

The time (in minutes) between arrivals of customers to a post office is to be modelled by the Exponential distribution with mean 0.33. Please give your answers to two decimal places.

Part a)

What is the probability that the time between consecutive customers is less than 15 seconds?

—

Part b)

Find the probability that the time between consecutive customers is between ten and fifteen seconds.

—

Part c)

Given that the time between consecutive customers arriving is greater than ten seconds, what is the chance that it is greater than fifteen seconds?

—

Solution:

Part a)

For $X \sim Exp(0.33)$, in general the cumulative distribution function is

$F(x) = 1 - e^{-x/0.33}$,

for x > 0. The probability can be found via

In R, we use

pexp(c(0.25), rate = 1/0.33, lower.tail=TRUE)

Part b)
The probability is

$P(1/6 < X < 1/4) = P(X < 1/4) - P(X < 1/6)$.

The first probability was found in part (a), and the second is

$F(1/6) = 1 - e^{-1/(6\times 0.33)}$.

The probability is 0.13.

Via R:

pexp(c(1/6), rate = 1/0.33, lower.tail=TRUE)

Part c)
Since $P(X > x) = e^{-x/0.33}$, we have

(or more directly by the lack of memory property).

Correct Answers:

- 0.53
- 0.13
- 0.78

+6pc-1pc
Problem 34

The manager of a supermarket tracked the amount of time needed for customers to be served by the cashier. After checking with his statistics professor, he concluded that the checkout times are exponentially distributed with a mean of 5.5 minutes. What propotion of customers require more than 11 minutes to check out?

Proportion = ______
Correct Answers:

- 0.135335283236613

+6pc-1pc
Problem 35

The manager of a gas station has observed that the times required by drivers to fill their car's tank and pay are quite variable. In fact, the times are exponentially distributed with a mean of 8 minutes. What is the probability that a car can complete the transaction in less than 6 minutes?

Probability = ______
Correct Answers:

- 0.527633447258985

+6pc-1pc

Problem 36

Suppose that X is an exponentially distributed random variable with $\lambda = 0.45$. Find each of the following probabilities:

A. $P(X > 1) =$ ______

B. $P(X > 0.28) =$ ______

C. $P(X < 0.45) =$ ______

D. $P(0.32 < X < 2.43) =$ ______

Correct Answers:

- 0.637628151621773
- 0.881614846783416
- 0.183313517401889
- 0.530845955154868

+6pc-1pc

Problem 37

Suppose that the time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 0.2$. What is

(a) the probability that a repair time exceeds 7 hours? ________

(b) the conditional probability that a repair takes at least 6 hours, given that it takes more than 5 hours? ________

Correct Answers:

- 0.246596963941606
- 0.818730753077982

+6pc-1pc

Problem 38

The annual salaries (in $) within a certain profession are modelled by a random variable with the cumulative distribution function

$$F(x) = \begin{cases} 1 - kx^{-3} \text{ for } x > 52000 \\ 0 \text{ otherwise,} \end{cases}$$

for some constant k. For these problems, please ensure your answers are accurate to within 3 decimals.

Part a)

Find the constant k here and provide its natural logarithm to three decimal places.

Natural logarithm of k: ___

Part b)

Calculate the mean salary given by the model.

___

Part c)

Find the proportion in the profession earning less than the mean, giving your answers as a fraction or to three decimal places.

___

Solution:

Part a)

As 52000 is the minimum salary, $F(52000)$ must be zero, so

$$F(52000) = 1 - k \cdot 52000^{-3} = 0$$

which implies that $k = 52000^3$ and that $\ln(k) = 32.577$.

Part b)
To find the p.d.f. of the salary variable X, we differentiate the c.d.f.,

for $x \geq 52000$. The mean salary is therefore

Part c)

The proportion of the workforce earning less than the mean is

Correct Answers:

- 32.577
- 78000
- 0.704

+6pc-1pc
Problem 39

The time, in 100 hours, that a student uses her game console over a year is a random variable X with probability density function

$$f(x) = \begin{cases} x \text{ if } 0 < x < 1 \\ 2 - x \text{ if } 1 \leq x < 2 \\ 0 \text{ otherwise.} \end{cases}$$

The power (in number of kilowatt hours) expended by the student's game console each year is $44X^2 + 22$. For these problems, please ensure your answers are accurate to within 3 decimals.

Part a)

Find the mean amount of power expended by the student's game console per year.

___

Part b)

Find the variance of power expended by the student's game console per year.

___

Solution:

Part a)

We require the expectation of X which is found by symmetry or via

Also required is $E(X^2)$, which is

With power denoted Y,

Part b)
We are required to find

Now by definition of the variance of a random variable,

$Var(X^2) = E(X^4) - E(X^2)^2$.

For the above we require

and so

$Var(Y) = 44^2(\frac{31}{15} - (\frac{7}{6})^2) = 1365.956$.

Correct Answers:

- 73.333
- 1365.96